CHAPTER 2

Basic Components and Electric Circuits

KEY CONCEPTS

- Basic Electrical Quantities and Associated Units: Charge, Current, Voltage, and Power
- Current Direction and Voltage Polarity
- The Passive Sign Convention for Calculating Power
- Ideal Voltage and Current Sources
- Dependent Sources
- Resistance and Ohm's Law

INTRODUCTION

In conducting circuit analysis, we often find ourselves seeking specific *currents, voltages,* or *powers,* so here we begin with a brief description of these quantities. In terms of components that can be used to build electrical circuits, we have quite a few from which to choose. We initially focus on the *resistor,* a simple passive component, and a range of idealized active sources of voltage and current. As we move forward, new components will be added to the inventory to allow more complex (and useful) circuits to be considered.

A quick word of advice before we begin: Pay close attention to the role of "+" and "−" signs when labeling voltages, and the significance of the arrow in defining current; they often make the difference between wrong and right answers.

2.1 UNITS AND SCALES

In order to state the value of some measurable quantity, we must give both a *number* and a *unit,* such as "3 meters." Fortunately, we all use the same number system. This is not true for units, and a little time must be spent in becoming familiar with a suitable system. We must agree on a standard unit and be assured of its permanence and its general acceptability. The standard unit of length, for example, should not be defined in terms of the distance between two marks on a certain rubber band; this is not permanent, and furthermore everybody else is using another standard.

The most frequently used system of units is the one adopted by the National Bureau of Standards in 1964; it is used by all major professional engineering societies and is the language in which today's textbooks are written. This is the International System of Units (abbreviated ***SI*** in all languages), adopted by the General

Conference on Weights and Measures in 1960. Modified several times since, the SI is built upon seven basic units: the *meter, kilogram, second, ampere, kelvin, mole,* and *candela* (see Table 2.1). This is a "metric system," some form of which is now in common use in most countries of the world, although it is not yet widely used in the United States. Units for other quantities such as volume, force, energy, etc., are derived from these seven base units.

There is some inconsistency regarding whether units named after a person should be capitalized. Here, we will adopt the most contemporary convention,[1,2] where such units are written out in lowercase (e.g., watt, joule), but abbreviated with an uppercase symbol (e.g., W, J).

(1) H. Barrell, *Nature* **220,** 1968, p. 651.
(2) V. N. Krutikov, T. K. Kanishcheva, S. A. Kononogov, L. K. Isaev, and N. I. Khanov, *Measurement Techniques* **51,** 2008, p. 1045.

TABLE 2.1 SI Base Units

Base Quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

The "calorie" used with food, drink, and exercise is really a kilocalorie, 4.187 J.

The fundamental unit of work or energy is the ***joule*** (J). One joule (a kg $m^2\,s^{-2}$ in SI base units) is equivalent to 0.7376 foot pound-force (ft · lbf). Other energy units include the calorie (cal), equal to 4.187 J; the British thermal unit (Btu), which is 1055 J; and the kilowatthour (kWh), equal to 3.6×10^6 J. Power is defined as the *rate* at which work is done or energy is expended. The fundamental unit of power is the ***watt*** (W), defined as 1 J/s. One watt is equivalent to 0.7376 ft · lbf/s or, equivalently, 1/745.7 horsepower (hp).

The SI uses the decimal system to relate larger and smaller units to the basic unit, and employs prefixes to signify the various powers of 10. A list of prefixes and their symbols is given in Table 2.2; the ones most commonly encountered in engineering are highlighted.

TABLE 2.2 SI Prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{-24}	yocto	y	10^{24}	yotta	Y
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-18}	atto	a	10^{18}	exa	E
10^{-15}	femto	f	10^{15}	peta	P
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{9}	giga	G
10^{-6}	micro	μ	10^{6}	mega	M
10^{-3}	milli	m	10^{3}	kilo	k
10^{-2}	centi	c	10^{2}	hecto	h
10^{-1}	deci	d	10^{1}	deka	da

These prefixes are worth memorizing, for they will appear often both in this text and in other technical work. Combinations of several prefixes, such as the millimicrosecond, are unacceptable. It is worth noting that in terms of distance, it is common to see "micron (μm)" as opposed to "micrometer," and often the angstrom (Å) is used for 10^{-10} meter. Also, in circuit analysis and engineering in general, it is fairly common to see numbers expressed in what are frequently termed "engineering units." In engineering notation, a quantity is represented by a number between 1 and 999 and an appropriate metric unit using a power divisible by 3. So, for example, it is preferable to express the quantity 0.048 W as 48 mW, instead of 4.8 cW, 4.8×10^{-2} W, or 48,000 μW.

PRACTICE

2.1 A krypton fluoride laser emits light at a wavelength of 248 nm. This is the same as: (*a*) 0.0248 mm; (*b*) 2.48 μm; (*c*) 0.248 μm; (*d*) 24,800 Å.

2.2 A single logic gate in a prototype integrated circuit is found to be capable of switching from the "on" state to the "off" state in 12 ps. This corresponds to: (*a*) 1.2 ns; (*b*) 120 ns; (*c*) 1200 ns; (*d*) 12,000 ns.

2.3 A typical incandescent reading lamp runs at 60 W. If it is left on constantly, how much energy (J) is consumed per day, and what is the weekly cost if energy is charged at a rate of 12.5 cents per kilowatthour?

Ans: 2.1 (c); 2.2 (d); 2.3 5.18 MJ, $1.26.

2.2 CHARGE, CURRENT, VOLTAGE, AND POWER

Charge

One of the most fundamental concepts in electric circuit analysis is that of charge conservation. We know from basic physics that there are two types of charge: positive (corresponding to a proton) and negative (corresponding to an electron). For the most part, this text is concerned with circuits in which only electron flow is relevant. There are many devices (such as batteries, diodes, and transistors) in which positive charge motion is important to understanding internal operation, but external to the device we typically concentrate on the electrons which flow through the connecting wires. Although we continuously transfer charges between different parts of a circuit, we do nothing to change the total amount of charge. In other words, we neither create nor destroy electrons (or protons) when running electric circuits.[1] Charge in motion represents a *current*.

In the SI system, the fundamental unit of charge is the ***coulomb*** (C). It is defined in terms of the ***ampere*** by counting the total charge that passes through an arbitrary cross section of a wire during an interval of one second; one coulomb is measured each second for a wire carrying a current of 1 ampere (Fig. 2.1). In this system of units, a single electron has a charge of -1.602×10^{-19} C and a single proton has a charge of $+1.602 \times 10^{-19}$ C.

(1) Although the occasional appearance of smoke may seem to suggest otherwise. . .

As seen in Table 2.1, the base units of the SI are not derived from fundamental physical quantities. Instead, they represent historically agreed upon measurements, leading to definitions which occasionally seem backward. For example, it would make more sense physically to define the ampere based on electronic charge.

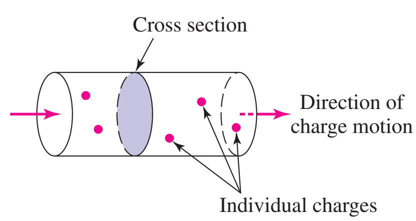


FIGURE 2.1 The definition of current illustrated using current flowing through a wire; 1 ampere corresponds to 1 coulomb of charge passing through the arbitrarily chosen cross section in 1 second.

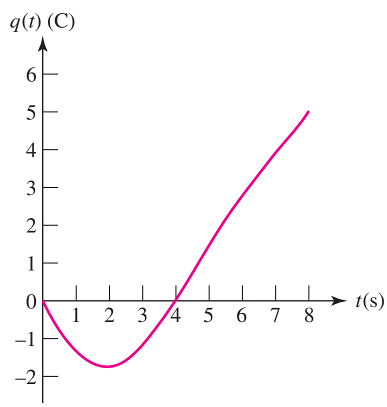
A quantity of charge that does not change with time is typically represented by Q. The instantaneous amount of charge (which may or may not be time-invariant) is commonly represented by $q(t)$, or simply q. This convention is used throughout the remainder of the text: capital letters are reserved for constant (time-invariant) quantities, whereas lowercase letters represent the more general case. Thus, a constant charge may be represented by *either* Q or q, but an amount of charge that changes over time *must* be represented by the lowercase letter q.

Current

The idea of "transfer of charge" or "charge in motion" is of vital importance to us in studying electric circuits because, in moving a charge from place to place, we may also transfer energy from one point to another. The familiar cross-country power-transmission line is a practical example of a device that transfers energy. Of equal importance is the possibility of varying the rate at which the charge is transferred in order to communicate or transfer information. This process is the basis of communication systems such as radio, television, and telemetry.

The current present in a discrete path, such as a metallic wire, has both a *numerical value* and a *direction* associated with it; it is a measure of the rate at which charge is moving past a given reference point in a specified direction.

Once we have specified a reference direction, we may then let $q(t)$ be the total charge that has passed the reference point since an arbitrary time $t = 0$, moving in the defined direction. A contribution to this total charge will be negative if negative charge is moving in the reference direction, or if positive charge is moving in the opposite direction. As an example, Fig. 2.2 shows a history of the total charge $q(t)$ that has passed a given reference point in a wire (such as the one shown in Fig. 2.1).



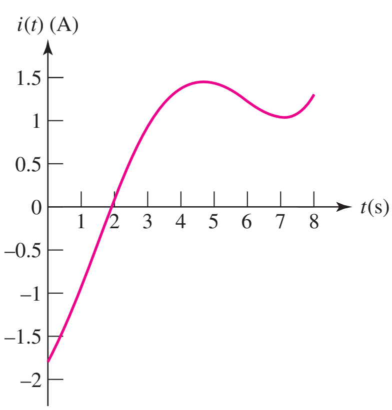
■ FIGURE 2.2 A graph of the instantaneous value of the total charge $q(t)$ that has passed a given reference point since $t = 0$.

We define the current at a specific point and flowing in a specified direction as the instantaneous rate at which net positive charge is moving past that point in the specified direction. This, unfortunately, is the historical definition, which came into popular use before it was appreciated that current in wires is actually due to negative, not positive, charge motion. Current is symbolized by I or i, and so

$$i = \frac{dq}{dt} \qquad [1]$$

The unit of current is the ampere (A), named after A. M. Ampère, a French physicist. It is commonly abbreviated as an "amp," although this is unofficial and somewhat informal. One ampere equals 1 coulomb per second.

Using Eq. [1], we compute the instantaneous current and obtain Fig. 2.3. The use of the lowercase letter i is again to be associated with an instantaneous value; an uppercase I would denote a constant (i.e., time-invariant) quantity.



■ FIGURE 2.3 The instantaneous current $i = dq/dt$, where q is given in Fig. 2.2.

The charge transferred between time t_0 and t may be expressed as a definite integral:

$$\int_{q(t_0)}^{q(t)} dq = \int_{t_0}^{t} i\, dt'$$

The total charge transferred over all time is thus given by

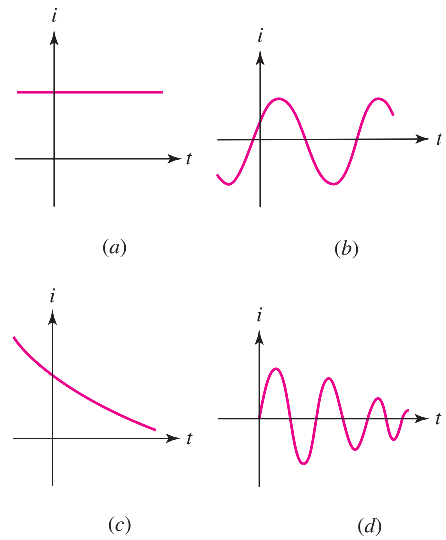
$$q(t) = \int_{t_0}^{t} i\, dt' + q(t_0) \qquad [2]$$

Several different types of current are illustrated in Fig. 2.4. A current that is constant in time is termed a direct current, or simply dc, and is shown by Fig. 2.4*a*. We will find many practical examples of currents that vary sinusoidally with time (Fig. 2.4*b*); currents of this form are present in normal household circuits. Such a current is often referred to as alternating current, or ac. Exponential currents and damped sinusoidal currents (Fig. 2.4*c* and *d*) will also be encountered later.

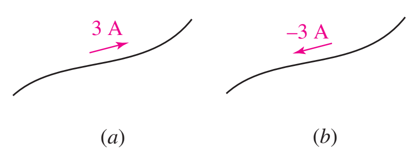
We create a graphical symbol for current by placing an arrow next to the conductor. Thus, in Fig. 2.5*a* the direction of the arrow and the value 3 A indicate either that a net positive charge of 3 C/s is moving to the right or that a net negative charge of −3 C/s is moving to the left each second. In Fig. 2.5*b* there are again two possibilities: either −3 A is flowing to the left or +3 A is flowing to the right. All four statements and both figures represent currents that are equivalent in their electrical effects, and we say that they are equal. A nonelectrical analogy that may be easier to visualize is to think in terms of a personal savings account: e.g., a deposit can be viewed as either a *negative* cash flow *out of* your account or a *positive* flow *into* your account.

It is convenient to think of current as the motion of positive charge, even though it is known that current flow in metallic conductors results from electron motion. In ionized gases, in electrolytic solutions, and in some semiconductor materials, however, positive charges in motion constitute part or all of the current. Thus, any definition of current can agree with the physical nature of conduction only part of the time. The definition and symbolism we have adopted are standard.

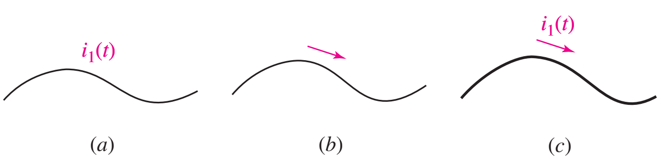
It is essential that we realize that the current arrow does not indicate the "actual" direction of current flow but is simply part of a convention that allows us to talk about "the current in the wire" in an unambiguous manner. The arrow is a fundamental part of the definition of a current! Thus, to talk about the value of a current $i_1(t)$ without specifying the arrow is to discuss an undefined entity. For example, Fig. 2.6*a* and *b* are meaningless representations of $i_1(t)$, whereas Fig. 2.6*c* is complete.



■ FIGURE 2.4 Several types of current: (*a*) Direct current (dc). (*b*) Sinusoidal current (ac). (*c*) Exponential current. (*d*) Damped sinusoidal current.



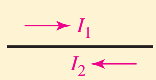
■ FIGURE 2.5 Two methods of representation for the exact same current.



■ FIGURE 2.6 (*a, b*) Incomplete, improper, and incorrect definitions of a current. (*c*) The correct definition of $i_1(t)$.

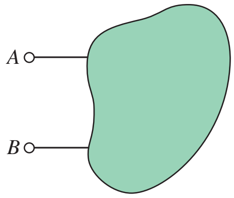
PRACTICE

2.4 In the wire of Fig. 2.7, electrons are moving *left* to *right* to create a current of 1 mA. Determine I_1 and I_2.

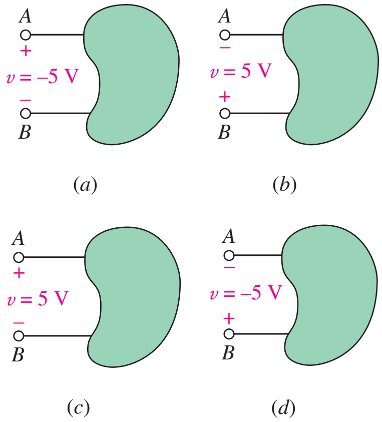


■ FIGURE 2.7

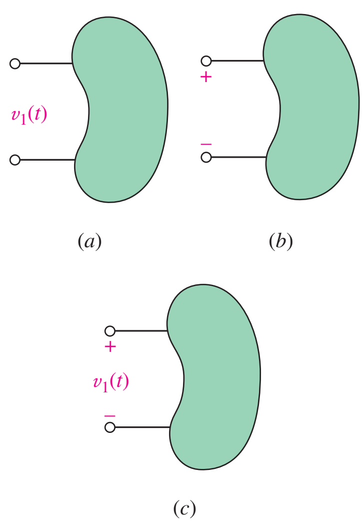
Ans: $I_1 = -1$ mA; $I_2 = +1$ mA.



■ FIGURE 2.8 A general two-terminal circuit element.



■ FIGURE 2.9 (*a*, *b*) Terminal *B* is 5 V positive with respect to terminal *A*; (*c*, *d*) terminal *A* is 5 V positive with respect to terminal *B*.



■ FIGURE 2.10 (*a, b*) These are inadequate definitions of a voltage. (*c*) A correct definition includes both a symbol for the variable and a plus-minus symbol pair.

Voltage

We must now begin to refer to a circuit element, something best defined in general terms to begin with. Such electrical devices as fuses, light bulbs, resistors, batteries, capacitors, generators, and spark coils can be represented by combinations of simple circuit elements. We begin by showing a very general circuit element as a shapeless object possessing two terminals at which connections to other elements may be made (Fig. 2.8).

There are two paths by which current may enter or leave the element. In subsequent discussions we will define particular circuit elements by describing the electrical characteristics that may be observed at their terminals.

In Fig. 2.8, let us suppose that a dc current is sent into terminal *A*, through the general element, and back out of terminal *B*. Let us also assume that pushing charge through the element requires an expenditure of energy. We then say that an electrical voltage (or a *potential difference*) exists between the two terminals, or that there is a voltage "across" the element. Thus, the voltage across a terminal pair is a measure of the work required to move charge through the element. The unit of voltage is the volt,[2] and 1 volt is the same as 1 J/C. Voltage is represented by V or v.

A voltage can exist between a pair of electrical terminals whether a current is flowing or not. An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals.

According to the principle of conservation of energy, the energy that is expended in forcing charge through the element must appear somewhere else. When we later meet specific circuit elements, we will note whether that energy is stored in some form that is readily available as electric energy or whether it changes irreversibly into heat, acoustic energy, or some other nonelectrical form.

We must now establish a convention by which we can distinguish between energy supplied *to* an element and energy that is supplied *by* the element itself. We do this by our choice of sign for the voltage of terminal *A* with respect to terminal *B*. If a positive current is entering terminal *A* of the element and an external source must expend energy to establish this current, then terminal *A* is positive with respect to terminal *B*. (Alternatively, we may say that terminal *B* is negative with respect to terminal *A*.)

The sense of the voltage is indicated by a plus-minus pair of algebraic signs. In Fig. 2.9*a*, for example, the placement of the + sign at terminal *A* indicates that terminal *A* is v volts positive with respect to terminal *B*. If we later find that v happens to have a numerical value of −5 V, then we may say either that *A* is −5 V positive with respect to *B* or that *B* is 5 V positive with respect to *A*. Other cases are shown in Fig. 2.9*b*, *c*, and *d*.

Just as we noted in our definition of current, it is essential to realize that the plus-minus pair of algebraic signs does not indicate the "actual" polarity of the voltage but is simply part of a convention that enables us to talk unambiguously about "the voltage across the terminal pair." *The definition of any voltage must include a plus-minus sign pair*! Using a quantity $v_1(t)$ without specifying the location of the plus-minus sign pair is using an undefined term. Figure 2.10*a* and *b* do *not* serve as definitions of $v_1(t)$; Fig. 2.10*c* does.

(2) We are probably fortunate that the full name of the 18th century Italian physicist, *Alessandro Giuseppe Antonio Anastasio Volta,* is not used for our unit of potential difference!

PRACTICE

2.5 For the element in Fig. 2.11, $v_1 = 17$ V. Determine v_2.

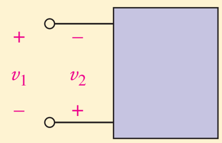


FIGURE 2.11

Ans: $v_2 = -17$ V.

Power

We have already defined power, and we will represent it by P or p. If one joule of energy is expended in transferring one coulomb of charge through the device in one second, then the rate of energy transfer is one watt. The absorbed power must be proportional both to the number of coulombs transferred per second (current) and to the energy needed to transfer one coulomb through the element (voltage). Thus,

$$p = vi \tag{3}$$

Dimensionally, the right side of this equation is the product of joules per coulomb and coulombs per second, which produces the expected dimension of joules per second, or watts. The conventions for current, voltage, and power are shown in Fig. 2.12.

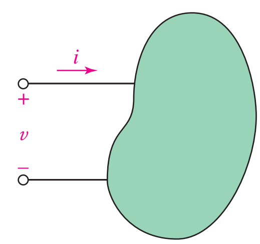


FIGURE 2.12 The power absorbed by the element is given by the product $p = vi$. Alternatively, we can say that the element generates or supplies a power $-vi$.

We now have an expression for the power being absorbed by a circuit element in terms of a voltage across it and current through it. Voltage was defined in terms of an energy expenditure, and power is the rate at which energy is expended. However, no statement can be made concerning energy transfer in any of the four cases shown in Fig. 2.9, for example, until the direction of the current is specified. Let us imagine that a current arrow is placed alongside each upper lead, directed to the right, and labeled "+2 A." First, consider the case shown in Fig. 2.9*c*. Terminal *A* is 5 V positive with respect to terminal *B*, which means that 5 J of energy is required to move each coulomb of positive charge into terminal *A*, through the object, and out terminal *B*. Since we are injecting +2 A (a current of 2 coulombs of positive charge per second) into terminal *A*, we are doing (5 J/C) × (2 C/s) = 10 J of work per second on the object. In other words, the object is absorbing 10 W of power from whatever is injecting the current.

We know from an earlier discussion that there is no difference between Fig. 2.9*c* and Fig. 2.9*d*, so we expect the object depicted in Fig. 2.9*d* to also be absorbing 10 W. We can check this easily enough: we are injecting +2 A into terminal *A* of the object, so +2 A flows out of terminal *B*. Another way of saying this is that we are injecting −2 A of current into terminal *B*. It takes −5 J/C to move charge from terminal *B* to terminal *A*, so the object is absorbing (−5 J/C) × (−2 C/s) = +10 W as expected. The only difficulty in describing this particular case is keeping the minus signs straight, but with a bit of care we see the correct answer can be obtained regardless of our choice of positive reference terminal (terminal *A* in Fig. 2.9*c*, and terminal *B* in Fig. 2.9*d*).

Now let's look at the situation depicted in Fig. 2.9*a*, again with $+2$ A injected into terminal *A*. Since it takes -5 J/C to move charge from terminal *A* to terminal *B*, the object is absorbing $(-5 \text{ J/C}) \times (2 \text{ C/s}) = -10$ W. What does this mean? How can anything absorb ***negative*** power? If we think about this in terms of energy transfer, -10 J is transferred to the object each second through the 2 A current flowing into terminal *A*. The object is actually losing energy—at a rate of 10 J/s. In other words, it is supplying 10 J/s (i.e., 10 W) to some other object not shown in the figure. Negative *absorbed* power, then, is equivalent to positive *supplied* power.

Let's recap. Figure 2.12 shows that if one terminal of the element is v volts positive with respect to the other terminal, and if a current i is entering the element through that terminal, then a power $p = vi$ is being *absorbed* by the element; it is also correct to say that a power $p = vi$ is being *delivered* to the element. When the current arrow is directed into the element at the plus-marked terminal, we satisfy the ***passive sign convention.*** This convention should be studied carefully, understood, and memorized. In other words, it says that if the current arrow and the voltage polarity signs are placed such that the current enters that end of the element marked with the positive sign, then the power *absorbed* by the element can be expressed by the product of the specified current and voltage variables. If the numerical value of the product is negative, then we say that the element is absorbing negative power, or that it is actually generating power and delivering it to some external element. For example, in Fig. 2.12 with $v = 5$ V and $i = -4$ A, the element may be described as either absorbing -20 W or generating 20 W.

If the current arrow is directed into the "$+$" marked terminal of an element, then $p = vi$ yields the *absorbed* power. A negative value indicates that power is actually being generated by the element.

If the current arrow is directed out of the "$+$" terminal of an element, then $p = vi$ yields the *supplied* power. A negative value in this case indicates that power is being absorbed.

Conventions are only required when there is more than one way to do something, and confusion may result when two different groups try to communicate. For example, it is rather arbitrary to always place "North" at the top of a map; compass needles don't point "up," anyway. Still, if we were talking to people who had secretly chosen the opposite convention of placing "South" at the top of their maps, imagine the confusion that could result! In the same fashion, there is a general convention that always draws the current arrows pointing into the positive voltage terminal, regardless of whether the element supplies or absorbs power. This convention is not incorrect but sometimes results in counterintuitive currents labeled on circuit schematics. The reason for this is that it simply seems more natural to refer to positive current flowing out of a voltage or current source that is supplying positive power to one or more circuit elements.

EXAMPLE 2.1

Compute the power absorbed by each part in Fig. 2.13.

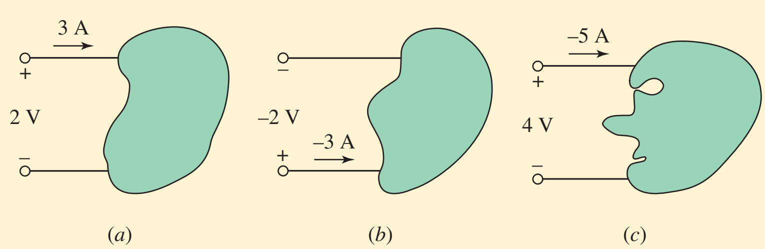


FIGURE 2.13 (*a, b, c*) Three examples of two-terminal elements.

In Fig. 2.13*a*, we see that the reference current is defined consistent with the passive sign convention, which assumes that the element is absorbing power. With +3 A flowing into the positive reference terminal, we compute

$$P = (2\text{ V})(3\text{ A}) = 6\text{ W}$$

of power absorbed by the element.

Figure 2.13*b* shows a slightly different picture. Now, we have a current of −3 A flowing into the positive reference terminal. This gives us an absorbed power

$$P = (-2\text{ V})(-3\text{ A}) = 6\text{ W}$$

Thus, we see that the two cases are actually equivalent: A current of +3 A flowing into the top terminal is the same as a current of +3 A flowing out of the bottom terminal, or, equivalently, a current of −3 A flowing into the bottom terminal.

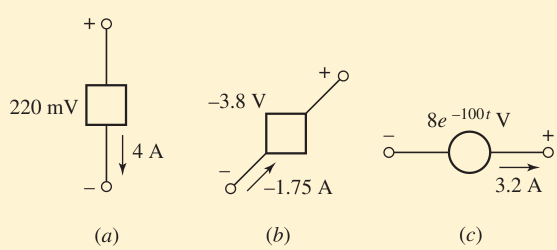
Referring to Fig. 2.13*c*, we again apply the passive sign convention rules and compute an absorbed power

$$P = (4\text{ V})(-5\text{ A}) = -20\text{ W}$$

Since we computed a negative *absorbed* power, this tells us that the element in Fig. 2.13*c* is actually *supplying* +20 W (i.e., it's a source of energy).

PRACTICE

2.6 Determine the power being absorbed by the circuit element in Fig. 2.14*a*.



■ FIGURE 2.14

2.7 Determine the power being generated by the circuit element in Fig. 2.14*b*.

2.8 Determine the power being delivered to the circuit element in Fig. 2.14*c* at $t = 5$ ms.

Ans: 880 mW; 6.65 W; −15.53 W.

2.3 VOLTAGE AND CURRENT SOURCES

Using the concepts of current and voltage, it is now possible to be more specific in defining a ***circuit element.***

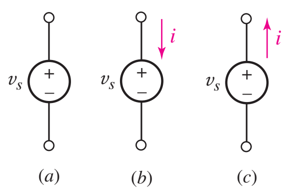
In so doing, it is important to differentiate between the physical device itself and the mathematical model which we will use to analyze its behavior in a circuit. The model is only an approximation.

Let us agree that we will use the expression *circuit element* to refer to the mathematical model. The choice of a particular model for any real device must be made on the basis of experimental data or experience; we will usually assume that this choice has already been made. For simplicity, we initially consider circuits with idealized components represented by simple models.

By definition, a simple circuit element is the mathematical model of a two-terminal electrical device, and it can be completely characterized by its voltage-current relationship; it cannot be subdivided into other two-terminal devices.

All the simple circuit elements that we will consider can be classified according to the relationship of the current through the element to the voltage across the element. For example, if the voltage across the element is linearly proportional to the current through it, we will call the element a resistor. Other types of simple circuit elements have terminal voltages which are proportional to the *derivative* of the current with respect to time (an inductor), or to the *integral* of the current with respect to time (a capacitor). There are also elements in which the voltage is completely independent of the current, or the current is completely independent of the voltage; these are termed *independent sources*. Furthermore, we will need to define special kinds of sources for which either the source voltage or current depends upon a current or voltage elsewhere in the circuit; such sources are referred to as *dependent sources*. Dependent sources are used a great deal in electronics to model both dc and ac behavior of transistors, especially in amplifier circuits.

Independent Voltage Sources



■ FIGURE 2.15 Circuit symbol of the independent voltage source.

The first element we will consider is the ***independent voltage source.*** The circuit symbol is shown in Fig. 2.15*a*; the subscript *s* merely identifies the voltage as a "source" voltage, and is common but not required. *An independent voltage source is characterized by a terminal voltage which is completely independent of the current through it.* Thus, if we are given an independent voltage source and are notified that the terminal voltage is 12 V, then we always assume this voltage, regardless of the current flowing.

The independent voltage source is an *ideal* source and does not represent exactly any real physical device, because the ideal source could theoretically deliver an infinite amount of energy from its terminals. This idealized voltage source does, however, furnish a reasonable approximation to several practical voltage sources. An automobile storage battery, for example, has a 12 V terminal voltage that remains essentially constant as long as the current through it does not exceed a few amperes. A small current may flow in either direction through the battery. If it is positive and flowing out of the positively marked terminal, then the battery is furnishing power to the headlights, for example; if the current is positive and flowing into the positive terminal, then the battery is charging by absorbing energy from the alternator.[3] An ordinary household electrical outlet also approximates an independent voltage source, providing a voltage $v_s = 115\sqrt{2}\cos 2\pi 60t$ V; this representation is valid for currents less than 20 A or so.

If you've ever noticed the room lights dim when an air conditioner kicks on, it's because the sudden large current demand temporarily led to a voltage drop. After the motor starts moving, it takes less current to keep it in motion. At that point, the current demand is reduced, the voltage returns to its original value, and the wall outlet again provides a reasonable approximation of an ideal voltage source.

A point worth repeating here is that the presence of the plus sign at the upper end of the symbol for the independent voltage source in Fig. 2.15*a* does not necessarily mean that the upper terminal is numerically positive with respect to the lower terminal. Instead, it means that the upper terminal is v_s volts positive with respect to the lower. If at some instant v_s happens to be negative, then the upper terminal is actually negative with respect to the lower at that instant.

(3) Or the battery of a friend's car, if you accidentally left your headlights on. . .

Consider a current arrow labeled "i" placed adjacent to the upper conductor of the source as in Fig. 2.15*b*. The current i is entering the terminal at which the positive sign is located, the passive sign convention is satisfied, and the source thus *absorbs* power $p = v_s i$. More often than not, a source is expected to deliver power to a network and not to absorb it. Consequently, we might choose to direct the arrow as in Fig. 2.15*c* so that $v_s i$ will represent the power *delivered* by the source. Technically, either arrow direction may be chosen; whenever possible, we will adopt the convention of Fig. 2.15*c* in this text for voltage and current sources, which are not usually considered passive devices.

An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Fig. 2.16*a* and *b*. Note in Fig. 2.16*b* that when the physical plate structure of the battery is suggested, the longer plate is placed at the positive terminal; the plus and minus signs then represent redundant notation, but they are usually included anyway. For the sake of completeness, the symbol for an independent ac voltage source is shown in Fig. 2.16*c*.

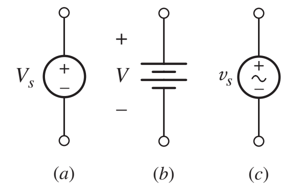


FIGURE 2.16 (*a*) DC voltage source symbol; (*b*) battery symbol; (*c*) ac voltage source symbol.

Terms like dc voltage source and dc current source are commonly used. Literally, they mean "direct-current voltage source" and "direct-current current source," respectively. Although these terms may seem a little odd or even redundant, the terminology is so widely used there's no point in fighting it.

Independent Current Sources

Another ideal source which we will need is the ***independent current source.*** Here, the current through the element is completely independent of the voltage across it. The symbol for an independent current source is shown in Fig. 2.17. If i_s is constant, we call the source an independent dc current source. An ac current source is often drawn with a tilde through the arrow, similar to the ac voltage source shown in Fig. 2.16*c*.

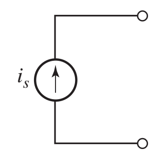


FIGURE 2.17 Circuit symbol for the independent current source.

Like the independent voltage source, the independent current source is at best a reasonable approximation for a physical element. In theory it can deliver infinite power from its terminals because it produces the same finite current for any voltage across it, no matter how large that voltage may be. It is, however, a good approximation for many practical sources, particularly in electronic circuits.

Although most students seem happy enough with an independent voltage source providing a fixed voltage but essentially any current, *it is a common mistake* to view an independent current source as having zero voltage across its terminals while providing a fixed current. In fact, we do not know a priori what the voltage across a current source will be—it depends entirely on the circuit to which it is connected.

Dependent Sources

The two types of ideal sources that we have discussed up to now are called *independent* sources because the value of the source quantity is not affected in any way by activities in the remainder of the circuit. This is in contrast with yet another kind of ideal source, the *dependent,* or *controlled,* source, in which the source quantity is determined by a voltage or current existing at some other location in the system being analyzed. Sources such as these appear in the equivalent electrical models for many electronic devices, such as transistors, operational amplifiers, and integrated circuits. To distinguish between dependent and independent sources, we introduce the diamond symbols shown in Fig. 2.18. In Fig. 2.18*a* and *c*, K is a dimensionless scaling constant. In Fig.2.18*b*, g is a scaling factor with units of A/V; in Fig. 2.18*d*, r is a scaling factor with units of V/A. The controlling current i_x and the controlling voltage v_x must be defined in the circuit.

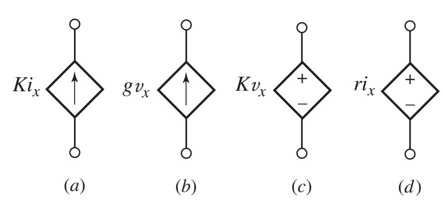


FIGURE 2.18 The four different types of dependent sources: (*a*) current-controlled current source; (*b*) voltage-controlled current source; (*c*) voltage-controlled voltage source; (*d*) current-controlled voltage source.

It does seem odd at first to have a current source whose value depends on a voltage, or a voltage source which is controlled by a current flowing through some other element. Even a voltage source depending on a remote voltage can appear strange. Such sources are invaluable for modeling complex systems, however, making the analysis algebraically straightforward. Examples include the drain current of a field effect transistor as a function of the gate voltage, or the output voltage of an analog integrated circuit as a function of differential input voltage. When encountered during circuit analysis, we write down the entire controlling expression for the dependent source just as we would if it was a numerical value attached to an independent source. This often results in the need for an additional equation to complete the analysis, unless the controlling voltage or current is already one of the specified unknowns in our system of equations.

EXAMPLE 2.2

In the circuit of Fig. 2.19*a*, if v_2 is known to be 3 V, find v_L.

We have been provided with a partially labeled circuit diagram and the additional information that $v_2 = 3$ V. This is probably worth adding to our diagram, as shown in Fig. 2.19*b*.

Next we step back and look at the information collected. In examining the circuit diagram, we notice that the desired voltage v_L is the same as the voltage across the dependent source. Thus,

$$v_L = 5v_2$$

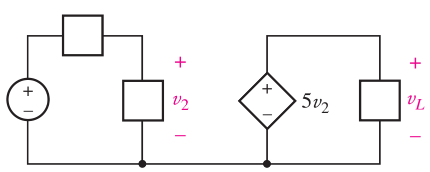
At this point, we would be done with the problem if only we knew v_2!

Returning to our diagram, we see that we actually do know v_2—it was specified as 3 V. We therefore write

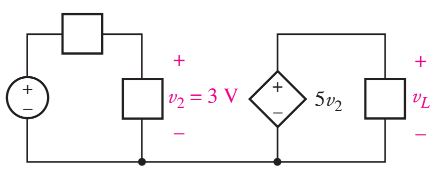
$$v_2 = 3$$

We now have two (simple) equations in two unknowns, and solve to find $v_L = 15$ V.

An important lesson at this early stage of the game is that *the time it takes to completely label a circuit diagram is always a good investment.* As a final step, we should go back and check over our work to ensure that the result is correct.



(*a*)

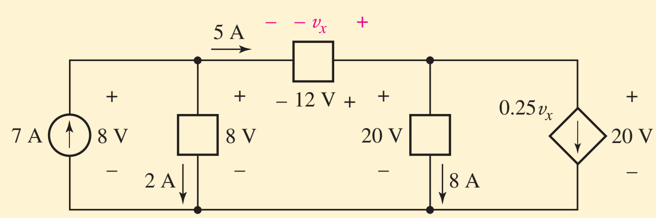


(*b*)

■ FIGURE 2.19 (*a*) An example circuit containing a voltage-controlled voltage source. (*b*) The additional information provided is included on the diagram.

PRACTICE

2.9 Find the power *absorbed* by each element in the circuit in Fig. 2.20.



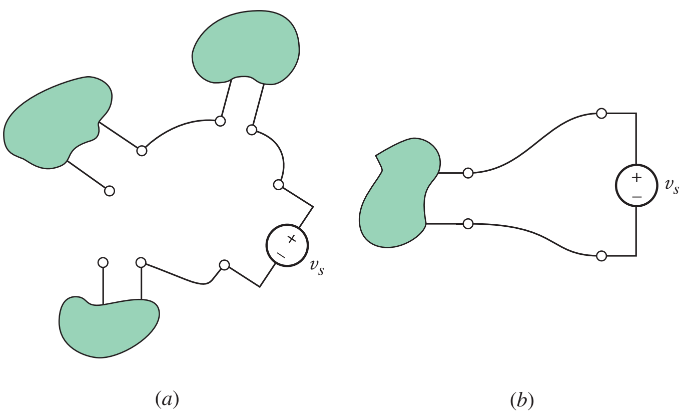
■ FIGURE 2.20

Ans: (left to right) −56 W; 16 W; −60 W; 160 W; −60 W.

Dependent and independent voltage and current sources are *active* elements; they are capable of delivering power to some external device. For the present we will think of a *passive* element as one which is capable only of receiving power. However, we will later see that several passive elements are able to store finite amounts of energy and then return that energy later to various external devices; since we still wish to call such elements passive, it will be necessary to improve upon our two definitions a little later.

Networks and Circuits

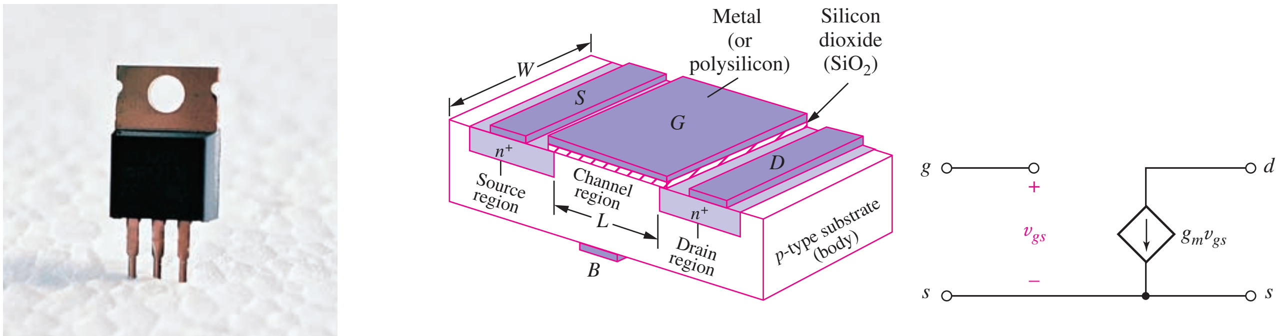
The interconnection of two or more simple circuit elements forms an electrical ***network.*** If the network contains at least one closed path, it is also an electric ***circuit.*** Note: Every circuit is a network, but not all networks are circuits (see Fig. 2.21)!



■ FIGURE 2.21 (*a*) A network that is not a circuit. (*b*) A network that is a circuit.

A network that contains at least one active element, such as an independent voltage or current source, is an active network. A network that does not contain any active elements is a passive network.

We have now defined what we mean by the term ***circuit element,*** and we have presented the definitions of several specific circuit elements, the independent and dependent voltage and current sources. Throughout the remainder of the book we will define only five additional circuit elements: the resistor, inductor, capacitor, transformer, and the ideal operational amplifier ("op amp," for short). These are all ideal elements. They are important because we may combine them into networks and circuits that represent real devices as accurately as we require. Thus, the transistor shown in Fig. 2.22*a* and *b* may be modeled by the voltage terminals designated v_{gs} and the single dependent current source of Fig. 2.22*c*. Note that the dependent current source produces a current that depends on a voltage elsewhere in the circuit. The parameter g_m, commonly referred to as the transconductance, is calculated using transistor-specific details as well as the operating point determined by the circuit connected to the transistor. It is generally a small number, on the order of 10^{-2} to perhaps 10 A/V. This model works pretty well as long as the frequency of any sinusoidal source is neither very large nor very small; the model can be modified to account for frequency-dependent



(*a*) (*b*) (*c*)

■ FIGURE 2.22 The Metal Oxide Semiconductor Field Effect Transistor (MOSFET). (*a*) An IRF540 N-channel power MOSFET in a TO-220 package, rated at 100 V and 22 A; (*b*) cross-sectional view of a basic MOSFET (*R. Jaeger, Microelectronic Circuit Design, McGraw-Hill, 1997*); (*c*) equivalent circuit model for use in ac circuit analysis.

effects by including additional ideal circuit elements such as resistors and capacitors.

Similar (but much smaller) transistors typically constitute only one small part of an integrated circuit that may be less than 2 mm × 2 mm square and 200 μm thick and yet contains several thousand transistors plus various resistors and capacitors. Thus, we may have a physical device that is about the size of one letter on this page but requires a model composed of ten thousand ideal simple circuit elements. We use this concept of "circuit modeling" in a number of electrical engineering topics covered in other courses, including electronics, energy conversion, and antennas.

2.4 OHM'S LAW

So far, we have been introduced to both dependent and independent voltage and current sources and were cautioned that they were *idealized* active elements that could only be approximated in a real circuit. We are now ready to meet another idealized element, the linear resistor. The resistor is the simplest passive element, and we begin our discussion by considering the work of an obscure German physicist, Georg Simon Ohm, who published a pamphlet in 1827 that described the results of one of the first efforts to measure currents and voltages, and to describe and relate them mathematically. One result was a statement of the fundamental relationship we now call ***Ohm's law,*** even though it has since been shown that this result was discovered 46 years earlier in England by Henry Cavendish, a brilliant semirecluse.

Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material, or

$$v = Ri \qquad [4]$$

where the constant of proportionality R is called the ***resistance.*** The unit of resistance is the *ohm,* which is 1 V/A and customarily abbreviated by a capital omega, Ω.

When this equation is plotted on i-versus-v axes, the graph is a straight line passing through the origin (Fig. 2.23). Equation [4] is a linear equation, and we will consider it as the definition of a *linear resistor*. Resistance is normally considered to be a positive quantity, although negative resistances may be simulated with special circuitry.

Again, it must be emphasized that the linear resistor is an idealized circuit element; it is only a mathematical model of a real, physical device. "Resistors" may be easily purchased or manufactured, but it is soon found that the voltage-current ratios of these physical devices are reasonably constant only within certain ranges of current, voltage, or power, and depend also on temperature and other environmental factors. We usually refer to a linear resistor as simply a resistor; any resistor that is nonlinear will always be described as such. Nonlinear resistors should not necessarily be considered undesirable elements. Although it is true that their presence complicates an analysis, the performance of the device may depend on or be greatly improved by the nonlinearity. For example, fuses for overcurrent protection and Zener diodes for voltage regulation are very nonlinear in nature, a fact that is exploited when using them in circuit design.

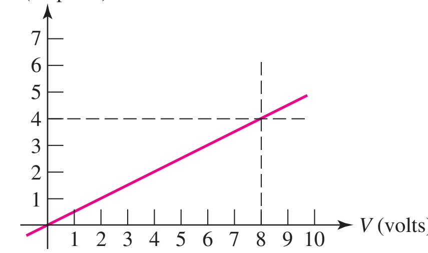
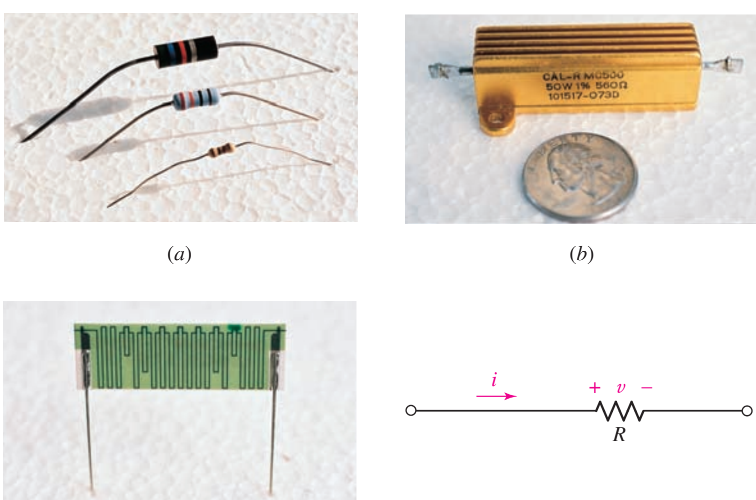


FIGURE 2.23 Current-voltage relationship for an example 2 Ω linear resistor. Note the slope of the line is 0.5 A/V, or 500 mΩ^{-1}.

Power Absorption

Figure 2.24 shows several different resistor packages, as well as the most common circuit symbol used for a resistor. In accordance with the voltage, current, and power conventions already adopted, the product of v and i gives the power absorbed by the resistor. That is, v and i are selected to satisfy the passive sign convention. The absorbed power appears physically



(*a*) (*b*) (*c*) (*d*)

FIGURE 2.24 (*a*) Several common resistor packages. (*b*) A 560 Ω power resistor rated at up to 50 W. (*c*) A 5% tolerance 10-teraohm (10,000,000,000,000 Ω) resistor manufactured by Ohmcraft. (*d*) Circuit symbol for the resistor, applicable to all of the devices in (*a*) through (*c*).

as heat and/or light and is always positive; a (positive) resistor is a passive element that cannot deliver power or store energy. Alternative expressions for the absorbed power are

$$p = vi = i^2R = v^2/R \qquad [5]$$

One of the authors (who shall remain anonymous) had the unfortunate experience of inadvertently connecting a 100 Ω, 2 W carbon resistor across a 110 V source. The ensuing flame, smoke, and fragmentation were rather disconcerting, demonstrating clearly that a practical resistor has definite limits to its ability to behave like the ideal linear model. In this case, the unfortunate resistor was called upon to absorb 121 W; since it was designed to handle only 2 W, its reaction was understandably violent.

EXAMPLE 2.3

The 560 Ω resistor shown in Fig. 2.24*b* is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

The voltage across the resistor is given by Ohm's law:

$$v = Ri = (560)(0.0424) = 23.7 \text{ V}$$

The dissipated power can be calculated in several different ways. For instance,

$$p = vi = (23.7)(0.0424) = 1.005 \text{ W}$$

Alternatively,

$$p = v^2/R = (23.7)^2/560 = 1.003 \text{ W}$$

or

$$p = i^2R = (0.0424)^2(560) = 1.007 \text{ W}$$

We note several things.

First, we calculated the power in three different ways, and we seem to have obtained *three different answers*!

In reality, however, we rounded our voltage to three significant digits, which will impact the accuracy of any subsequent quantity we calculate with it. With this in mind, we see that the answers show reasonable agreement (within 1%).

The other point worth noting is that the resistor is rated to 50 W—since we are only dissipating approximately 2% of this value, the resistor is in no danger of overheating.

PRACTICE

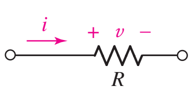


FIGURE 2.25

With reference to Fig. 2.25, compute the following:

2.10 R if $i = -2\ \mu$A and $v = -44$ V.

2.11 The power absorbed by the resistor if $v = 1$ V and $R = 2$ kΩ.

2.12 The power absorbed by the resistor if $i = 3$ nA and $R = 4.7$ MΩ.

Ans: 22 MΩ; 500 μW; 42.3 pW.

PRACTICAL APPLICATION

Wire Gauge

Technically speaking, any material (except for a superconductor) will provide resistance to current flow. As in all introductory circuits texts, however, we tacitly assume that wires appearing in circuit diagrams have zero resistance. This implies that there is no potential difference between the ends of a wire, and hence no power absorbed or heat generated. Although usually not an unreasonable assumption, it does neglect practical considerations when choosing the appropriate wire diameter for a specific application.

Resistance is determined by (1) the inherent resistivity of a material and (2) the device geometry. ***Resistivity,*** represented by the symbol ρ, is a measure of the ease with which electrons can travel through a certain material. Since it is the ratio of the electric field (V/m) to the areal density of current flowing in the material (A/m^2), the general unit of ρ is an $\Omega \cdot$m, although metric prefixes are often employed. Every material has a different inherent resistivity, which depends on temperature. Some examples are shown in Table 2.3; as can be seen, there is a small variation between different types of copper (less than 1%) but a very large difference between different metals. In particular, although physically stronger than copper, steel wire is several times more resistive. In some technical discussions, it is more common to see the conductivity (symbolized by σ) of a material quoted, which is simply the reciprocal of the resistivity.

The resistance of a particular object is obtained by multiplying the resistivity by the length ℓ of the resistor and dividing by the cross-sectional area (A) as in Eq. [6]; these parameters are illustrated in Fig. 2.26.

$$R = \rho \frac{\ell}{A} \qquad [6]$$

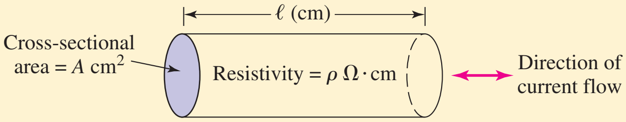


FIGURE 2.26 Definition of geometrical parameters used to compute the resistance of a wire. The resistivity of the material is assumed to be spatially uniform.

We determine the resistivity when we select the material from which to fabricate a wire and measure the temperature of the application environment. Since a finite amount of power is absorbed by the wire due to its resistance, current flow leads to the production of heat. Thicker wires have lower resistance and also dissipate heat more easily but are heavier, take up a larger volume, and are more expensive. Thus, we are motivated by practical considerations to choose the smallest wire that

TABLE 2.3 Common Electrical Wire Materials and Resistivities*

ASTM Specification**	Temper and Shape	Resistivity at 20°C ($\mu\Omega \cdot$cm)
B33	Copper, tinned soft, round	1.7654
B75	Copper, tube, soft, OF copper	1.7241
B188	Copper, hard bus tube, rectangular or square	1.7521
B189	Copper, lead-coated soft, round	1.7654
B230	Aluminum, hard, round	2.8625
B227	Copper-clad steel, hard, round, grade 40 HS	4.3971
B355	Copper, nickel-coated soft, round Class 10	1.9592
B415	Aluminum-clad steel, hard, round	8.4805

* C. B. Rawlins, "Conductor materials," *Standard Handbook for Electrical Engineering,* 13th ed., D. G. Fink and H. W. Beaty, eds. New York: McGraw-Hill, 1993, pp. 4-4 to 4-8.

** American Society of Testing and Materials.

(Continued on next page)

can safely do the job, rather than simply choosing the largest-diameter wire available in an effort to minimize resistive losses. The American Wire Gauge (AWG) is a standard system of specifying wire size. In selecting a wire gauge, smaller AWG corresponds to a larger wire diameter; an abbreviated table of common gauges is given in Table 2.4. Local fire and electrical safety codes typically dictate the required gauge for specific wiring applications, based on the maximum current expected as well as where the wires will be located.

TABLE 2.4 Some Common Wire Gauges and the Resistance of (Soft) Solid Copper Wire*

Conductor Size (AWG)	Cross-Sectional Area (mm^2)	Ohms per 1000 ft at 20°C
28	0.0804	65.3
24	0.205	25.7
22	0.324	16.2
18	0.823	6.39
14	2.08	2.52
12	3.31	1.59
6	13.3	0.3952
4	21.1	0.2485
2	33.6	0.1563

* C. B. Rawlins et al., *Standard Handbook for Electrical Engineering,* 13th ed., D. G. Fink and H. W. Beaty, eds. New York: McGraw-Hill, 1993, p. 4-47.

EXAMPLE 2.4

A dc power link is to be made between two islands separated by a distance of 24 miles. The operating voltage is 500 kV and the system capacity is 600 MW. Calculate the maximum dc current flow, and estimate the resistivity of the cable, assuming a diameter of 2.5 cm and a solid (not stranded) wire.

Dividing the maximum power (600 MW, or 600×10^6 W)
by the operating voltage (500 kV, or 500×10^3 V)
yields a maximum current of

$$\frac{600 \times 10^6}{500 \times 10^3} = 1200 \text{ A}$$

The cable resistance is simply the ratio of the voltage to the current, or

$$R_{\text{cable}} = \frac{500 \times 10^3}{1200} = 417\ \Omega$$

We know the length:

$$\ell = (24 \text{ miles})\left(\frac{5280 \text{ ft}}{1 \text{ mile}}\right)\left(\frac{12 \text{ in}}{1 \text{ ft}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right) = 3{,}862{,}426 \text{ cm}$$

Given that most of our information appears to be valid to only two significant figures, we round this to 3.9×10^6 cm.

With the cable diameter specified as 2.5 cm, we know its cross-sectional area is 4.9 cm^2.

Thus, $\rho_{\text{cable}} = R_{\text{cable}} \dfrac{A}{\ell} = 417 \left(\dfrac{4.9}{3.9 \times 10^6}\right) = 520\ \mu\Omega \cdot \text{cm}$

PRACTICE

2.13 A 500 ft long 24 AWG soft copper wire is carrying a current of 100 mA. What is the voltage dropped across the wire?

Ans: 3.26 V.

Conductance

For a linear resistor the ratio of current to voltage is also a constant

$$\frac{i}{v} = \frac{1}{R} = G \qquad [7]$$

where G is called the *conductance*. The SI unit of conductance is the siemens (S), 1 A/V. An older, unofficial unit for conductance is the mho, which was often abbreviated as ℧ and is still occasionally written as Ω^{-1}. You will occasionally see it used on some circuit diagrams, as well as in catalogs and texts. The same circuit symbol (Fig. 2.24*d*) is used to represent both resistance and conductance. The absorbed power is again necessarily positive and may be expressed in terms of the conductance by

$$p = vi = v^2 G = \frac{i^2}{G} \qquad [8]$$

Thus a 2 Ω resistor has a conductance of $\frac{1}{2}$ S, and if a current of 5 A is flowing through it, then a voltage of 10 V is present across the terminals and a power of 50 W is being absorbed.

All the expressions given so far in this section were written in terms of instantaneous current, voltage, and power, such as $v = iR$ and $p = vi$. We should recall that this is a shorthand notation for $v(t) = Ri(t)$ and $p(t) = v(t)\, i(t)$. The current through and voltage across a resistor must both vary with time in the same manner. Thus, if $R = 10\ \Omega$ and $v = 2 \sin 100t$ V, then $i = 0.2 \sin 100t$ A. Note that the power is given by $0.4 \sin^2 100t$ W, and a simple sketch will illustrate the different nature of its variation with time. Although the current and voltage are each negative during certain time intervals, the absorbed power is *never* negative!

Resistance may be used as the basis for defining two commonly used terms, *short circuit* and *open circuit*. We define a short circuit as a resistance of zero ohms; then, since $v = iR$, the voltage across a short circuit must be zero, although the current may have any value. In an analogous manner, we define an open circuit as an infinite resistance. It follows from Ohm's law that the current must be zero, regardless of the voltage across the open circuit. Although real wires have a small resistance associated with them, we always assume them to have zero resistance unless otherwise specified. Thus, in all of our circuit schematics, wires are taken to be perfect short circuits.

SUMMARY AND REVIEW

In this chapter, we introduced the topic of units – specifically those relevant to electrical circuits—and their relationship to fundamental (SI) units. We also discussed current and current sources, voltage and voltage sources, and the fact that the product of voltage and current yields power (the rate of energy consumption or generation). Since power can be either positive or negative depending on the current direction and voltage polarity, the passive sign convention was described to ensure we always know if an element is *absorbing* or *supplying* energy to the rest of the circuit. Four additional sources were introduced, forming a general class known as dependent sources. They are often used to model complex systems and electrical components, but the actual value of voltage or current supplied is typically unknown until the entire circuit is analyzed. We concluded the chapter with the resistor—by far the most common circuit element—whose voltage and current are linearly related (described by Ohm's law). Whereas the *resistivity* of a material is one of its fundamental properties (measured in $\Omega \cdot$ cm), *resistance* describes a device property (measured in Ω) and hence depends not only on resistivity but on the device geometry (i.e., length and area) as well.

We conclude with key points of this chapter to review, along with appropriate examples.

- ❑ The system of units most commonly used in electrical engineering is the SI.
- ❑ The direction in which positive charges are moving is the direction of positive current flow; alternatively, positive current flow is in the direction opposite that of moving electrons.
- ❑ To define a current, both a value and a direction must be given. Currents are typically denoted by the uppercase letter "I" for constant (dc) values, and either $i(t)$ or simply i otherwise.
- ❑ To define a voltage across an element, it is necessary to label the terminals with "+" and "−" signs as well as to provide a value (either an algebraic symbol or a numerical value).
- ❑ Any element is said to supply positive power if positive current flows out of the positive voltage terminal. Any element absorbs positive power if positive current flows into the positive voltage terminal. (Example 2.1)
- ❑ There are six sources: the independent voltage source, the independent current source, the current-controlled dependent current source, the voltage-controlled dependent current source, the voltage-controlled dependent voltage source, and the current-controlled dependent voltage source. (Example 2.2)

Note that a current represented by i or $i(t)$ can be constant (dc) or time-varying, but currents represented by the symbol I must be non-time-varying.

- Ohm's law states that the voltage across a linear resistor is directly proportional to the current flowing through it; i.e., $v = Ri$. (Example 2.3)
- The power dissipated by a resistor (which leads to the production of heat) is given by $p = vi = i^2R = v^2/R$. (Example 2.3)
- Wires are typically assumed to have zero resistance in circuit analysis. When selecting a wire gauge for a specific application, however, local electrical and fire codes must be consulted. (Example 2.4)

READING FURTHER

A good book that discusses the properties and manufacture of resistors in considerable depth:

Felix Zandman, Paul-René Simon, and Joseph Szwarc, *Resistor Theory and Technology.* Raleigh, N.C.: SciTech Publishing, 2002.

A good all-purpose electrical engineering handbook:

Donald G. Fink and H. Wayne Beaty, *Standard Handbook for Electrical Engineers,* 13th ed., New York: McGraw-Hill, 1993.

In particular, pp. 1-1 to 1-51, 2-8 to 2-10, and 4-2 to 4-207 provide an in-depth treatment of topics related to those discussed in this chapter.

A detailed reference for the SI is available on the Web from the National Institute of Standards:

Ambler Thompson and Barry N. Taylor, *Guide for the Use of the International System of Units (SI),* NIST Special Publication 811, 2008 edition, www.nist.gov.

EXERCISES

2.1 Units and Scales

1. Convert the following to engineering notation:
   (*a*) 0.045 W (*b*) 2000 pJ
   (*c*) 0.1 ns (*d*) 39,212 as
   (*e*) 3 Ω (*f*) 18,000 m
   (*g*) 2,500,000,000,000 bits (*h*) 10^{15} atoms/cm^3
2. Convert the following to engineering notation:
   (*a*) 1230 fs (*b*) 0.0001 decimeter
   (*c*) 1400 mK (*d*) 32 nm
   (*e*) 13,560 kHz (*f*) 2021 micromoles
   (*g*) 13 deciliters (*h*) 1 hectometer
3. Express the following in engineering units:
   (*a*) 1212 mV (*b*) 10^{11} pA
   (*c*) 1000 yoctoseconds (*d*) 33.9997 zeptoseconds
   (*e*) 13,100 attoseconds (*f*) 10^{-14} zettasecond
   (*g*) 10^{-5} second (*h*) 10^{-9} Gs
4. Expand the following distances in simple meters:
   (*a*) 1 Zm (*b*) 1 Em (*c*) 1 Pm
   (*d*) 1 Tm (*e*) 1 Gm (*f*) 1 Mm

5. Convert the following to SI units, taking care to employ proper engineering notation:
   (*a*) 212°F (*b*) 0°F (*c*) 0 K
   (*d*) 200 hp (*e*) 1 yard (*f*) 1 mile
6. Convert the following to SI units, taking care to employ proper engineering notation:
   (*a*) 100°C (*b*) 0°C (*c*) 4.2 K
   (*d*) 150 hp (*e*) 500 Btu (*f*) 100 J/s
7. A certain krypton fluoride laser generates 15 ns long pulses, each of which contains 550 mJ of energy. (*a*) Calculate the peak instantaneous output power of the laser. (*b*) If up to 100 pulses can be generated per second, calculate the maximum average power output of the laser.
8. When operated at a wavelength of 750 nm, a certain Ti:sapphire laser is capable of producing pulses as short as 50 fs, each with an energy content of 500 μJ. (*a*) Calculate the instantaneous output power of the laser. (*b*) If the laser is capable of a pulse repetition rate of 80 MHz, calculate the maximum average output power that can be achieved.
9. An electric vehicle is driven by a single motor rated at 40 hp. If the motor is run continuously for 3 h at maximum output, calculate the electrical energy consumed. Express your answer in SI units using engineering notation.
10. Under insolation conditions of 500 W/m^2 (direct sunlight), and 10% solar cell efficiency (defined as the ratio of electrical output power to incident solar power), calculate the area required for a photovoltaic (solar cell) array capable of running the vehicle in Exer. 9 at half power.
11. A certain metal oxide nanowire piezoelectricity generator is capable of producing 100 pW of usable electricity from the type of motion obtained from a person jogging at a moderate pace. (*a*) How many nanowire devices are required to operate a personal MP3 player which draws 1 W of power? (*b*) If the nanowires can be produced with a density of 5 devices per square micron directly onto a piece of fabric, what area is required, and would it be practical?
12. A particular electric utility charges customers different rates depending on their daily rate of energy consumption: \$0.05/kWh up to 20 kWh, and \$0.10/kWh for all energy usage above 20 kWh in any 24 hour period. (*a*) Calculate how many 100 W light bulbs can be run continuously for less than \$10 per week. (*b*) Calculate the daily energy cost if 2000 kW of power is used continuously.
13. The Tilting Windmill Electrical Cooperative LLC Inc. has instituted a differential pricing scheme aimed at encouraging customers to conserve electricity use during daylight hours, when local business demand is at its highest. If the price per kilowatthour is \$0.033 between the hours of 9 p.m. and 6 a.m., and \$0.057 for all other times, how much does it cost to run a 2.5 kW portable heater continuously for 30 days?
14. Assuming a global population of 9 billion people, each using approximately 100 W of power continuously throughout the day, calculate the total land area that would have to be set aside for photovoltaic power generation, assuming 800 W/m^2 of incident solar power and a conversion efficiency (sunlight to electricity) of 10%.

2.2 Charge, Current, Voltage, and Power

15. The total charge flowing out of one end of a small copper wire and into an unknown device is determined to follow the relationship $q(t) = 5e^{-t/2}$ C, where t is expressed in seconds. Calculate the current flowing into the device, taking note of the sign.
16. The current flowing into the collector lead of a certain bipolar junction transistor (BJT) is measured to be 1 nA. If no charge was transferred in or out of the collector lead prior to $t = 0$, and the current flows for 1 min, calculate the total charge which crosses into the collector.

17. The total charge stored on a 1 cm diameter insulating plate is -10^{13} C. (*a*) How many electrons are on the plate? (*b*) What is the areal density of electrons (number of electrons per square meter)? (*c*) If additional electrons are added to the plate from an external source at the rate of 10^6 electrons per second, what is the magnitude of the current flowing between the source and the plate?
18. A mysterious device found in a forgotten laboratory accumulates charge at a rate specified by the expression $q(t) = 9 - 10t$ C from the moment it is switched on. (*a*) Calculate the total charge contained in the device at $t = 0$. (*b*) Calculate the total charge contained at $t = 1$ s. (*c*) Determine the current flowing into the device at $t = 1$ s, 3 s, and 10 s.
19. A new type of device appears to accumulate charge according to the expression $q(t) = 10t^2 - 22t$ mC (t in s). (*a*) In the interval $0 \le t < 5$ s, at what time does the current flowing into the device equal zero? (*b*) Sketch $q(t)$ and $i(t)$ over the interval $0 \le t < 5$ s.
20. The current flowing through a tungsten-filament light bulb is determined to follow $i(t) = 114 \sin(100\pi t)$ A. (*a*) Over the interval defined by $t = 0$ and $t = 2$ s, how many times does the current equal zero amperes? (*b*) How much charge is transported through the light bulb in the first second?
21. The current waveform depicted in Fig. 2.27 is characterized by a period of 8 s. (*a*) What is the average value of the current over a single period? (*b*) If $q(0) = 0$, sketch $q(t)$, $0 < t < 20$ s.

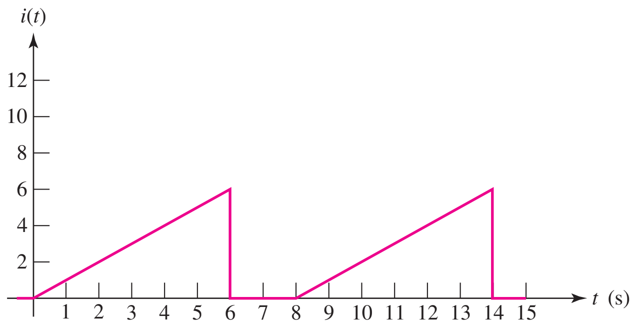


FIGURE 2.27 An example of a time-varying current.

22. The current waveform depicted in Fig. 2.28 is characterized by a period of 4 s. (*a*) What is the average value of the current over a single period? (*b*) Compute the average current over the interval $1 < t < 3$ s. (*c*) If $q(0) = 1$ C, sketch $q(t)$, $0 < t < 4$ s.

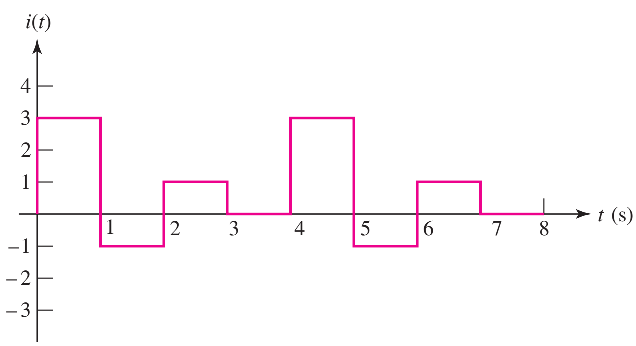
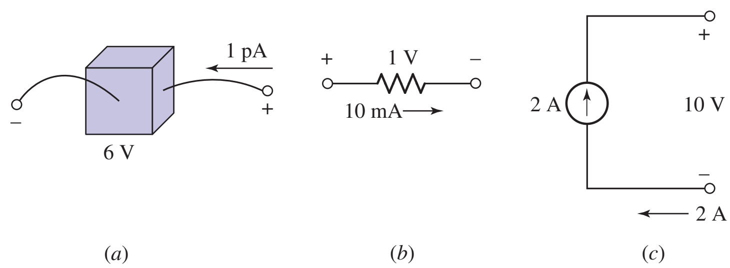


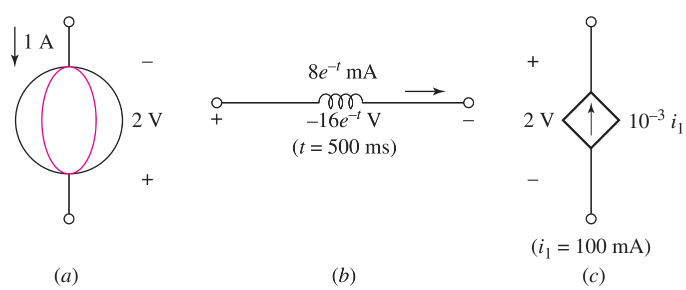
FIGURE 2.28 An example of a time-varying current.

23. A path around a certain electric circuit has discrete points labeled A, B, C, and D. To move an electron from points A to C requires 5 pJ. To move an electron from B to C requires 3 pJ. To move an electron from A to D requires 8 pJ. (a) What is the potential difference (in volts) between points B and C, assuming a "+" reference at C? (b) What is the potential difference (in volts) between points B and D, assuming a "+" reference at D? (c) What is the potential difference (in volts) between points A and B (again, in volts), assuming a "+" reference at B?
24. Two metallic terminals protrude from a device. The terminal on the left is the positive reference for a voltage called v_x (the other terminal is the negative reference). The terminal on the right is the positive reference for a voltage called v_y (the other terminal being the negative reference). If it takes 1 mJ of energy to push a single electron into the left terminal, determine the voltages v_x and v_y.
25. The convention for voltmeters is to use a black wire for the negative reference terminal and a red wire for the positive reference terminal. (a) Explain why two wires are required to measure a voltage. (b) If it is dark and the wires into the voltmeter are swapped by accident, what will happen during the next measurement?
26. Determine the power absorbed by each of the elements in Fig. 2.29.



■ FIGURE 2.29 Elements for Exer. 26.

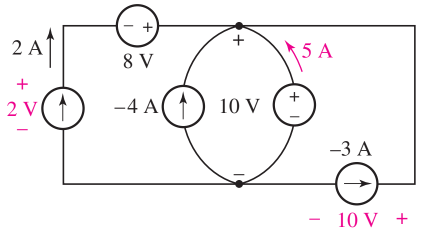
27. Determine the power absorbed by each of the elements in Fig. 2.30.



■ FIGURE 2.30 Elements for Exer. 27.

28. A constant current of 1 ampere is measured flowing into the positive reference terminal of a pair of leads whose voltage we'll call v_p. Calculate the absorbed power at $t = 1$ s if $v_p(t)$ equals (a) +1 V; (b) −1 V; (c) $2 + 5\cos(5t)$ V; (d) $4e^{-2t}$ V, (e) Explain the significance of a negative value for absorbed power.

29. Determine the power supplied by the leftmost element in the circuit of Fig. 2.31.



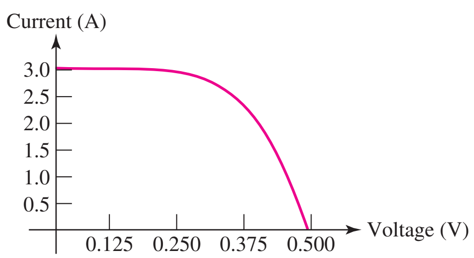
■ FIGURE 2.31

30. The current-voltage characteristic of a silicon solar cell exposed to direct sunlight at noon in Florida during midsummer is given in Fig. 2.32. It is obtained by placing different-sized resistors across the two terminals of the device and measuring the resulting currents and voltages.

(*a*) What is the value of the short-circuit current?

(*b*) What is the value of the voltage at open circuit?

(*c*) Estimate the maximum power that can be obtained from the device.



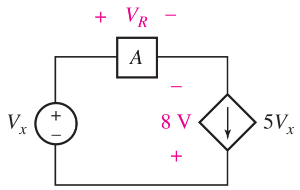
■ FIGURE 2.32

2.3 Voltage and Current Sources

31. Some of the ideal sources in the circuit of Fig. 2.31 are supplying positive power, and others are absorbing positive power. Determine which are which, and show that the algebraic sum of the power absorbed by each element (taking care to preserve signs) is equal to zero.

32. By careful measurements it is determined that a benchtop argon ion laser is consuming (absorbing) 1.5 kW of electric power from the wall outlet, but only producing 5 W of optical power. Where is the remaining power going? Doesn't conservation of energy require the two quantities to be equal?

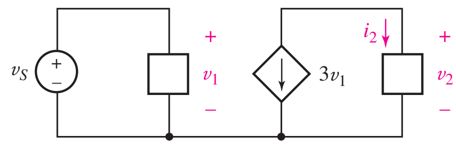
33. Refer to the circuit represented in Fig. 2.33, while noting that the same current flows through each element. The voltage-controlled dependent source provides a current which is 5 times as large as the voltage V_x. (a) For $V_R = 10$ V and $V_x = 2$ V, determine the power absorbed by each element. (*b*) Is element *A* likely a passive or active source? Explain.



■ FIGURE 2.33

34. Refer to the circuit represented in Fig. 2.33, while noting that the same current flows through each element. The voltage-controlled dependent source provides a current which is 5 times as large as the voltage V_x. (*a*) For $V_R = 100$ V and $V_x = 92$ V, determine the power supplied by each element. (*b*) Verify that the algebraic sum of the supplied powers is equal to zero.

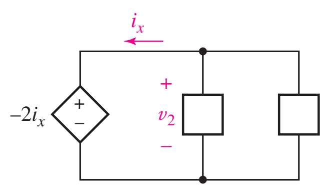
35. The circuit depicted in Fig. 2.34 contains a dependent current source; the magnitude and direction of the current it supplies are directly determined by the voltage labeled v_1. Note that therefore $i_2 = -3v_1$. Determine the voltage v_1 if $v_2 = 33i_2$ and $i_2 = 100$ mA.



■ FIGURE 2.34

36. To protect an expensive circuit component from being delivered too much power, you decide to incorporate a fast-blowing fuse into the design. Knowing that the circuit component is connected to 12 V, its minimum power consumption is 12 W, and the maximum power it can safely dissipate is 100 W, which of the three available fuse ratings should you select: 1 A, 4 A, or 10 A? Explain your answer.

37. The dependent source in the circuit of Fig. 2.35 provides a voltage whose value depends on the current i_x. What value of i_x is required for the dependent source to be supplying 1 W?



■ FIGURE 2.35

2.4 Ohm's Law

38. Determine the magnitude of the current flowing through a 4.7 kΩ resistor if the voltage across it is (*a*) 1 mV; (*b*) 10 V; (*c*) $4e^{-t}$ V; (*d*) $100\cos(5t)$ V; (*e*) −7 V.

39. Real resistors can only be manufactured to a specific tolerance, so that in effect the value of the resistance is uncertain. For example, a 1 Ω resistor specified as 5% tolerance could in practice be found to have a value anywhere in the range of 0.95 to 1.05 Ω. Calculate the voltage across a 2.2 kΩ 10% tolerance resistor if the current flowing through the element is (*a*) 1 mA; (*b*) $4\sin 44t$ mA.

40. (*a*) Sketch the current-voltage relationship (current on the *y*-axis) of a 2 kΩ resistor over the voltage range of $-10\text{ V} \le V_{\text{resistor}} \le +10$ V. Be sure to label both axes appropriately. (*b*) What is the numerical value of the slope (express your answer in siemens)?

41. Sketch the voltage across a 33 Ω resistor over the range $0 < t < 2\pi$ s, if the current is given by $2.8\cos(t)$ A. Assume both the current and voltage are defined according to the passive sign convention.

42. Figure 2.36 depicts the current-voltage characteristic of three different resistive elements. Determine the resistance of each, assuming the voltage and current are defined in accordance with the passive sign convention.

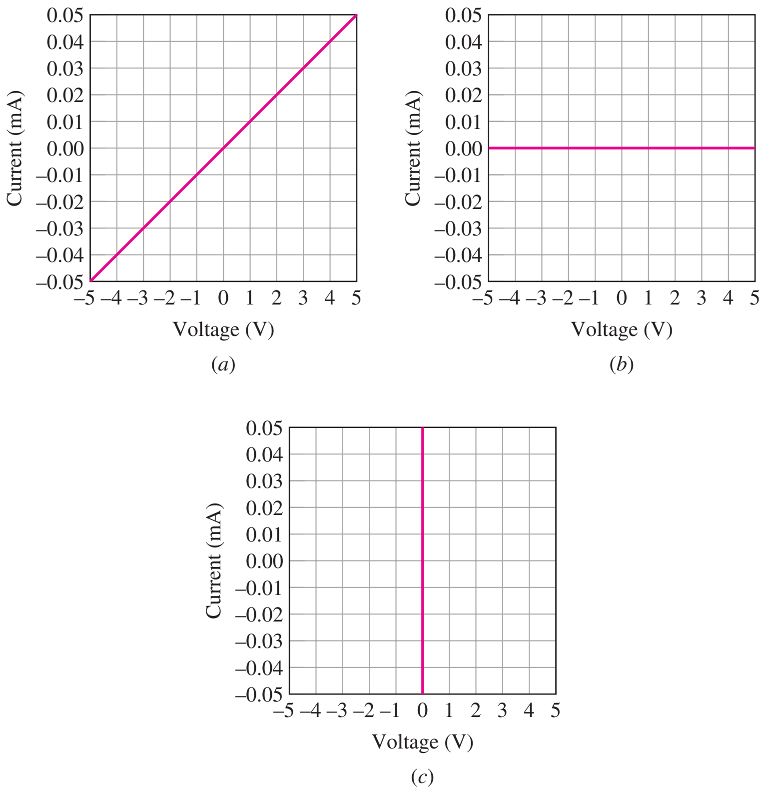


FIGURE 2.36

43. Determine the conductance (in siemens) of the following: (*a*) 0 Ω; (*b*) 100 MΩ; (*c*) 200 mΩ.

44. Determine the magnitude of the current flowing through a 10 mS conductance if the voltage across it is (*a*) 2 mV; (*b*) −1 V; (*c*) $100e^{-2t}$ V; (*d*) $5 \sin(5t)$ V; (*e*) 0 V.

45. A 1% tolerance 1 kΩ resistor may in reality have a value anywhere in the range of 990 to 1010 Ω. Assuming a voltage of 9 V is applied across it, determine (*a*) the corresponding range of current and (*b*) the corresponding range of absorbed power. (*c*) If the resistor is replaced with a 10% tolerance 1 kΩ resistor, repeat parts (*a*) and (*b*).

46. The following experimental data is acquired for an unmarked resistor, using a variable-voltage power supply and a current meter. The current meter readout is somewhat unstable, unfortunately, which introduces error into the measurement.

Voltage (V)	Current (mA)
−2.0	−0.89
−1.2	−0.47
0.0	0.01
1.0	0.44
1.5	0.70

(*a*) Plot the measured current-versus-voltage characteristic.

(*b*) Using a best-fit line, estimate the value of the resistance.

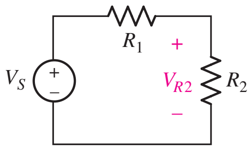


FIGURE 2.37

47. Utilize the fact that in the circuit of Fig. 2.37, the total power supplied by the voltage source must equal the total power absorbed by the two resistors to show that

$$V_{R_2} = V_S \frac{R_2}{R_1 + R_2}$$

You may assume the same current flows through each element (a requirement of charge conservation).

48. For each of the circuits in Fig. 2.38, find the current I and compute the power absorbed by the resistor.

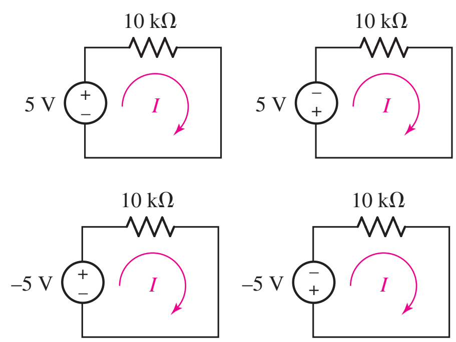


FIGURE 2.38

49. Sketch the power absorbed by a 100 Ω resistor as a function of voltage over the range $-2\text{ V} \leq V_{\text{resistor}} \leq +2\text{ V}$.

Chapter-Integrating Exercises

DP 50. So-called "n-type" silicon has a resistivity given by $\rho = (-qN_D\mu_n)^{-1}$, where N_D is the volume density of phosphorus atoms (atoms/cm^3), μ_n is the electron mobility (cm^2/V · s), and $q = -1.602 \times 10^{-19}$ C is the charge of each electron. Conveniently, a relationship exists between mobility and N_D, as shown in Fig. 2.39. Assume an 8 inch diameter silicon wafer (disk) having a thickness of 300 μm. Design a 10 Ω resistor by specifying a phosphorus concentration in the range of $2 \times 10^{15}\text{ cm}^{-3} \leq N_D \leq 2 \times 10^{17}\text{ cm}^{-3}$, along with a suitable geometry (the wafer may be cut, but not thinned).

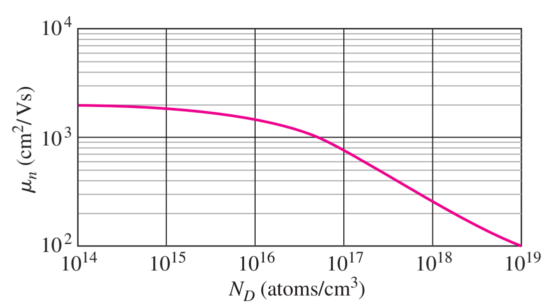


FIGURE 2.39

51. Figure 2.39 depicts the relationship between electron mobility μ_n and dopant density N_D for n-type silicon. With the knowledge that resistivity in this material is given by $\rho = N_D\mu_n/q$, plot resistivity as a function of dopant density over the range $10^{14}\text{ cm}^{-3} \leq N_D \leq 10^{19}\text{ cm}^{-3}$.

DP 52. Referring to the data of Table 2.4, design a resistor whose value can be varied mechanically in the range of 100 to 500 Ω (assume operation at 20°C).

53. A 250 ft long span separates a dc power supply from a lamp which draws 25 A of current. If 14 AWG wire is used (note that two wires are needed for a total of 500 ft), calculate the amount of power wasted in the wire.

54. The resistance values in Table 2.4 are calibrated for operation at 20°C. They may be corrected for operation at other temperatures using the relationship[4]

$$\frac{R_2}{R_1} = \frac{234.5 + T_2}{234.5 + T_1}$$

where T_1 = reference temperature (20°C in present case)
T_2 = desired operating temperature
R_1 = resistance at T_1
R_2 = resistance at T_2

A piece of equipment relies on an external wire made of 28 AWG soft copper, which has a resistance of 50.0 Ω at 20°C. Unfortunately, the operating environment has changed, and it is now 110.5°F. (*a*) Calculate the length of the original wire. (*b*) Determine by how much the wire should be shortened so that it is once again 50.0 Ω.

DP 55. Your favorite meter contains a precision (1% tolerance) 10 Ω resistor. Unfortunately, the last person who borrowed this meter somehow blew the resistor, and it needs to be replaced. Design a suitable replacement, assuming at least 1000 ft of each of the wire gauges listed in Table 2.4 is readily available to you.

56. At a new installation, you specified that all wiring should conform to the ASTM B33 specification (see Table 2.3). Unfortunately the subcontractor misread your instructions and installed B415 wiring instead (but the same gauge). Assuming the operating voltage is unchanged, (*a*) by how much will the current be reduced, and (*b*) how much additional power will be wasted in the lines? (*Express both answers in terms of percentage.*)

57. If 1 mA of current is forced through a 1 mm diameter, 2.3 meter long piece of hard, round, aluminum-clad steel (B415) wire, how much power is wasted as a result of resistive losses? If instead wire of the same dimensions but conforming to B75 specifications is used, by how much will the power wasted due to resistive losses be reduced?

58. The network shown in Fig. 2.40 can be used to accurately model the behavior of a bipolar junction transistor provided that it is operating in the forward active mode. The parameter β is known as the current gain. If for this device

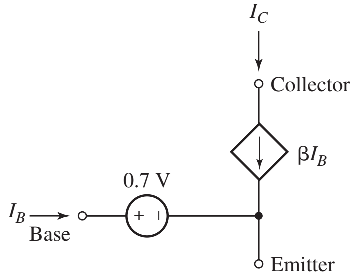


FIGURE 2.40 DC model for a bipolar junction transistor operating in forward active mode.

(4) D. G. Fink and H. W. Beaty, *Standard Handbook for Electrical Engineers,* 13th ed. New York: McGraw-Hill, 1993, p. 2–9.

$\beta = 100$, and I_B is determined to be 100 μA, calculate (*a*) I_C, the current flowing into the collector terminal; and (*b*) the power dissipated by the base-emitter region.

59. A 100 W tungsten filament light bulb functions by taking advantage of resistive losses in the filament, absorbing 100 joules each second of energy from the wall socket. How much *optical* energy per second do you expect it to produce, and does this violate the principle of energy conservation?

60. Batteries come in a wide variety of types and sizes. Two of the most common are called "AA" and "AAA." A single battery of either type is rated to produce a terminal voltage of 1.5 V when fully charged. So what are the differences between the two, other than size? (*Hint:* Think about energy.)