1 Circuit Concepts

Electric circuits, which are collections of *circuit elements* connected together, are the most fundamental structures of electrical engineering. A circuit is an interconnection of simple electrical devices that have at least one closed path in which current may flow. However, we may have to clarify to some of our readers what is meant by "current" and "electrical device," a task that we shall undertake shortly. Circuits are important in electrical engineering because they process electrical signals, which carry energy and information; a signal can be any time-varying electrical quantity. Engineering circuit analysis is a mathematical study of some useful interconnection of simple electrical devices. An electric circuit, as discussed in this book, is an idealized mathematical *model* of some physical circuit or phenomenon. The ideal circuit elements are the resistor, the inductor, the capacitor, and the voltage and current sources. The ideal circuit model helps us to *predict*, mathematically, the approximate behavior of the actual event. The models also provide insights into how to *design* a physical electric circuit to perform a desired task. Electrical engineering is concerned with the *analysis* and *design* of electric circuits, systems, and devices. In Chapter 1 we shall deal with the fundamental concepts that underlie all circuits.

Electrical quantities will be introduced first. Then the reader is directed to the lumped-circuit elements. Then Ohm's law and Kirchhoff's laws are presented. These laws are sufficient for analyzing and designing simple but illustrative practical circuits. Later, a brief introduction is given to meters and measurements. Finally, the analogy between electrical and other nonelectric physical systems is pointed out. The chapter ends with a case study of practical application.

1.1 ELECTRICAL QUANTITIES

In describing the operation of electric circuits, one should be familiar with such electrical quantities as charge, current, and voltage. The material of this section will serve as a review, since it will not be entirely new to most readers.

Charge and Electric Force

The proton has a charge of $+1.602\ 10^{-19}$ coulombs (C), while the electron has a charge of -1.602×10^{-19} C. The neutron has zero charge. Electric charge and, more so, its movement are the most basic items of interest in electrical engineering. When many charged particles are collected together, larger charges and charge distributions occur. There may be point charges (C), line charges (C/m), surface charge distributions (C/m^2), and volume charge distributions (C/m^3).

A charge is responsible for an *electric field* and charges exert *forces* on each other. Like charges repel, whereas unlike charges attract. Such an electric force can be controlled and utilized for some useful purpose. *Coulomb's law* gives an expression to evaluate the electric force in newtons (N) exerted on one point charge by the other:

$$\text{Force on } Q_1 \text{ due to } Q_2 = \bar{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\bar{a}_{21} \tag{1.1.1a}$$

$$\text{Force on } Q_2 \text{ due to } Q_1 = \bar{F}_{12} = \frac{Q_2 Q_1}{4\pi\varepsilon_0 R^2}\bar{a}_{12} \tag{1.1.1b}$$

where Q_1 and Q_2 are the point charges (C); R is the separation in meters (m) between them; ε_0 is the permittivity of the free-space medium with units of $C^2/N \cdot m$ or, more commonly, farads per meter (F/m); and $\bar{a}_{21}$ and $\bar{a}_{12}$ are unit vectors along the line joining Q_1 and Q_2, as shown in Figure 1.1.1.

Equation (1.1.1) shows the following:

1. Forces $\bar{F}_{21}$ and $\bar{F}_{12}$ are experienced by Q_1 and Q_2, due to the presence of Q_2 and Q_1, respectively. They are equal in magnitude and opposite of each other in direction.
2. The magnitude of the force is proportional to the product of the charge magnitudes.
3. The magnitude of the force is inversely proportional to the square of the distance between the charges.
4. The magnitude of the force depends on the medium.
5. The direction of the force is along the line joining the charges.

Note that the SI system of units will be used throughout this text, and the student should be conversant with the conversion factors for the SI system.

The force per unit charge experienced by a small test charge placed in an electric field is known as the electric field intensity $\bar{E}$, whose units are given by N/C or, more commonly, volts per meter (V/m),

$$\bar{E} = \lim_{Q \to 0} \frac{\bar{F}}{Q} \tag{1.1.2}$$

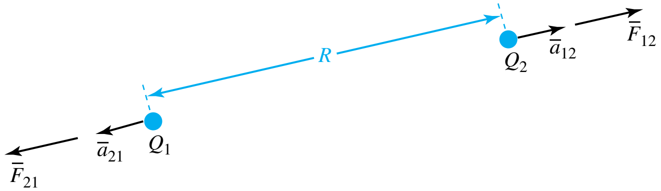


Figure 1.1.1 Illustration of Coulomb's law.

Equation (1.1.2) is the defining equation for the electric field intensity (with units of N/C or V/m), irrespective of the source of the electric field. One may then conclude:

$$\bar{F}_{21} = Q_1 \bar{E}_2 \tag{1.1.3a}$$

$$\bar{F}_{12} = Q_2 \bar{E}_1 \tag{1.1.3b}$$

where $\bar{E}_2$ is the electric field due to Q_2 at the location of Q_1, and $\bar{E}_1$ is the electric field due to Q_1 at the location of Q_2, given by

$$\bar{E}_2 = \frac{Q_2}{4\pi \varepsilon_0 R^2} \bar{a}_{21} \tag{1.1.4a}$$

$$\bar{E}_1 = \frac{Q_1}{4\pi \varepsilon_0 R^2} \bar{a}_{12} \tag{1.1.4b}$$

Note that the electric field intensity due to a positive point charge is directed everywhere radially away from the point charge, and its constant-magnitude surfaces are spherical surfaces centered at the point charge.

EXAMPLE 1.1.1

(a) A small region of an impure silicon crystal with dimensions 1.25×10^{-6} m $\times 10^{-3}$ m $\times 10^{-3}$ m has only the ions (with charge $+1.6\ 10^{-19}$ C) present with a volume density of $10^{25}/\text{m}^3$. The rest of the crystal volume contains equal densities of electrons (with charge -1.6×10^{-19} C) and positive ions. Find the net total charge of the crystal.

(b) Consider the charge of part (a) as a point charge Q_1. Determine the force exerted by this on a charge $Q_2 = 3\mu$C when the charges are separated by a distance of 2 m in free space, as shown in Figure E1.1.1.

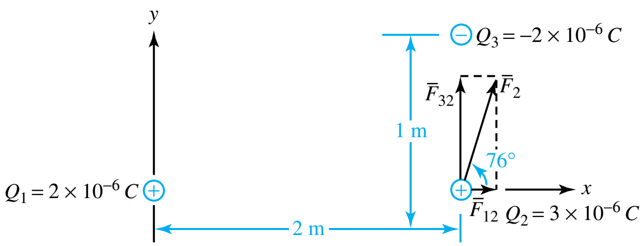


Figure E1.1.1

(c) If another charge $Q_3 = -2\mu$C is added to the system 1 m above Q_2, as shown in Figure E1.1.1, calculate the force exerted on Q_2.

Solution

(a) In the region where both ions and free electrons exist, their opposite charges cancel, and the net charge density is zero. From the region containing ions only, the volume-charge density is given by

$$\rho = (10^{25})(1.6 \times 10^{-19}) = 1.6 \times 10^6 \text{ C}/m^3$$

The net total charge is then calculated as:

$$Q = \rho v = (1.6 \times 10^6)(1.25 \times 10^{-6} \times 10^{-3} \times 10^{-3}) = 2 \times 10^{-6} \text{ C}$$

(b) The rectangular coordinate system shown defines the locations of the charges: $Q_1 = 2 \times 10^{-6}$ C; $Q_2 = 3 \times 10^{-6}$ C. The force that Q_1 exerts on Q_2 is in the positive direction of x, given by Equation (1.1.1),

$$\bar{F}_{12} = \frac{(3 \times 10^{-6})(2 \times 10^{-6})}{4\pi(10^{-9}/36\pi)2^2}\,\bar{a}_x = \bar{a}_x\, 13.5 \times 10^{-3} \text{ N}$$

This is the force experienced by Q_2 due to the effect of the electric field of Q_1. Note the value used for free-space permittivity, ε_0, as (8.854×10^{-12}), or approximately $10^{-9}/36\pi$ F/m. $\bar{a}_x$ is the unit vector in the positive x-direction.

(c) When Q_3 is added to the system, as shown in Figure E1.1.1, an additional force on Q_2 directed in the positive y-direction occurs (since Q_3 and Q_2 are of opposite sign),

$$\bar{F}_{32} = \frac{(3 \times 10^{-6})(-2 \times 10^{-6})}{4\pi(10^{-9}36\pi)1^2}(-\bar{a}_y) = \bar{a}_y\, 54 \times 10^{-3} \text{ N}$$

The resultant force $\bar{F}_2$ acting on Q_2 is the superposition of $\bar{F}_{12}$ and $\bar{F}_{32}$ due to Q_1 and Q_3, respectively.

The vector combination of $\bar{F}_{12}$ and $\bar{F}_{32}$ is given by:

$$\bar{F}_2 = \sqrt{F_{12}^2 + F_{32}^2}\,\angle \tan^{-1}\frac{\bar{F}_{32}}{\bar{F}_{12}}$$

$$= \sqrt{13.5^2 + 54^2} \times 10^{-3} \angle \tan^{-1}\frac{54}{13.5}$$

$$= 55.7 \times 10^{-3} \angle 76^\circ \text{ N}$$

Conductors and Insulators

In order to put charge in motion so that it becomes an electric current, one must provide a path through which it can flow easily by the movement of electrons. Materials through which charge flows readily are called *conductors*. Examples include most metals, such as silver, gold, copper, and aluminum. Copper is used extensively for the conductive paths on electric circuit boards and for the fabrication of electrical wires.

Insulators are materials that do not allow charge to move easily. Examples include glass, plastic, ceramics, and rubber. Electric current cannot be made to flow through an insulator, since a charge has great difficulty moving through it. One sees insulating (or *dielectric*) materials often wrapped around the center conducting core of a wire.

Although the term resistance will be formally defined later, one can say qualitatively that a conductor has a very low resistance to the flow of charge, whereas an insulator has a very high resistance to the flow of charge. Charge-conducting abilities of various materials vary in a wide range. *Semiconductors* fall in the middle between conductors and insulators, and have a moderate resistance to the flow of charge. Examples include silicon, germanium, and gallium arsenide.

Current and Magnetic Force

The rate of movement of net positive charge per unit of time through a cross section of a conductor is known as *current*,

$$i(t) = \frac{dq}{dt} \tag{1.1.5}$$

The SI unit of current is the ampere (A), which represents 1 coulomb per second. In most metallic conductors, such as copper wires, current is exclusively the movement of free electrons in the wire. Since electrons are negative, and since the direction designated for the current is that of the net positive charge movement, the charges in the wire are thus moving in the direction opposite to the direction of the current designation. The net charge transferred at a particular time is the net area under the current–time curve from the beginning of time to the present,

$$q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau \tag{1.1.6}$$

While Coulomb's law has to do with the electric force associated with two charged bodies, *Ampere's law of force* is concerned with magnetic forces associated with two loops of wire carrying currents by virtue of the motion of charges in the loops. Note that isolated current elements do not exist without sources and sinks of charges at their ends; magnetic monopoles do not exist. Figure 1.1.2 shows two loops of wire in freespace carrying currents I_1 and I_2.

Considering a differential element $d\bar{l}_1$ of loop 1 and a differential element $d\bar{l}_2$ of loop 2, the differential magnetic forces $d\bar{F}_{21}$ and $d\bar{F}_{12}$ experienced by the differential current elements $I_1\, d\bar{l}_1$, and $I_2\, d\bar{l}_2$, due to I_2 and I_1, respectively, are given by

$$d\bar{F}_{21} = I_1\, d\bar{l}_1 \times \left(\frac{\mu_0}{4\pi} \frac{I_2 d\bar{l}_2 \times \bar{a}_{21}}{R^2} \right) \tag{1.1.7a}$$

$$d\bar{F}_{12} = I_2\, d\bar{l}_2 \times \left(\frac{\mu_0}{4\pi} \frac{I_1 d\bar{l}_1 \times \bar{a}_{12}}{R^2} \right) \tag{1.1.7b}$$

where $\bar{a}_{21}$ and $\bar{a}_{12}$ are unit vectors along the line joining the two current elements, R is the distance between the centers of the elements, μ_0 is the permeability of free space with units of N/A^2 or commonly known as henrys per meter (H/m). Equation (1.1.7) reveals the following:

1. The magnitude of the force is proportional to the product of the two currents and the product of the lengths of the two current elements.

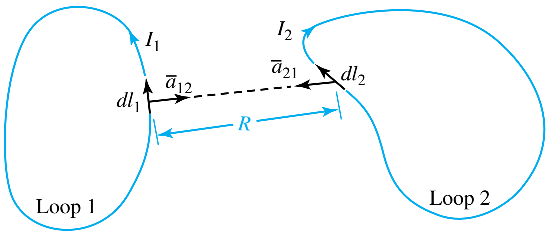


Figure 1.1.2 Illustration of Ampere's law (of force).

2. The magnitude of the force is inversely proportional to the square of the distance between the current elements.
3. To determine the direction of, say, the force acting on the current element $I_1\, d\bar{l}_1$, the *cross product* $d\bar{l}_2 \times \bar{a}_{21}$ must be found. Then crossing $d\bar{l}_1$ with the resulting vector will yield the direction of $d\bar{F}_{21}$.
4. Each current element is acted upon by a *magnetic field* due to the other current element,

$$d\bar{F}_{21} = I_1\, d\bar{l}_1 \times \bar{B}_2 \tag{1.1.8a}$$

$$d\bar{F}_{12} = I_2 d\bar{l}_2 \times \bar{B}_1 \tag{1.1.8b}$$

where $\bar{B}$ is known as the *magnetic flux density vector* with units of N/A · m, commonly known as webers per square meter (Wb/m^2) or tesla (T).

Current distribution is the source of magnetic field, just as charge distribution is the source of electric field. As a consequence of Equations (1.1.7) and (1.1.8), it can be seen that

$$\bar{B}_2 = \frac{\mu_0}{4\pi}\, I_2\, d\bar{l}_2 \times \bar{a}_{21} \tag{1.1.9a}$$

$$\bar{B}_1 = \frac{\mu_0}{4\pi}\, \frac{I_1\, d\bar{l}_1 \times \bar{a}_{12}}{R^2} \tag{1.1.9b}$$

which depend on the medium parameter. Equation (1.1.9) is known as the *Biot–Savart law*. Equation (1.1.8) can be expressed in terms of moving charge, since current is due to the flow of charges. With $I = dq/dt$ and $d\bar{l} = \bar{v}\, dt$, where $\bar{v}$ is the velocity, Equation (1.1.8) can be rewritten as

$$d\bar{F} = \left(\frac{dq}{dt}\right)(\bar{v}\, dt) \times \bar{B} = dq\,(\bar{v} \times \bar{B}) \tag{1.1.10}$$

Thus it follows that the force $\bar{F}$ experienced by a test charge q moving with a velocity $\bar{v}$ in a magnetic field of flux density $\bar{B}$ is given by

$$\bar{F} = q\,(\bar{v} \times \bar{B}) \tag{1.1.11}$$

The expression for the total force acting on a test charge q moving with velocity $\bar{v}$ in a region characterized by electric field intensity $\bar{E}$ and a magnetic field of flux density $\bar{B}$ is

$$\bar{F} = \bar{F}_E + \bar{F}_M = q\,(\bar{E} + \bar{v} \times \bar{B}) \tag{1.1.12}$$

which is known as the *Lorentz force equation*.

EXAMPLE 1.1.2

Figure E1.1.2 (a) gives a plot of $q(t)$ as a function of time t.

(a) Obtain the plot of $i(t)$.

(b) Find the average value of the current over the time interval of 1 to 7 seconds.

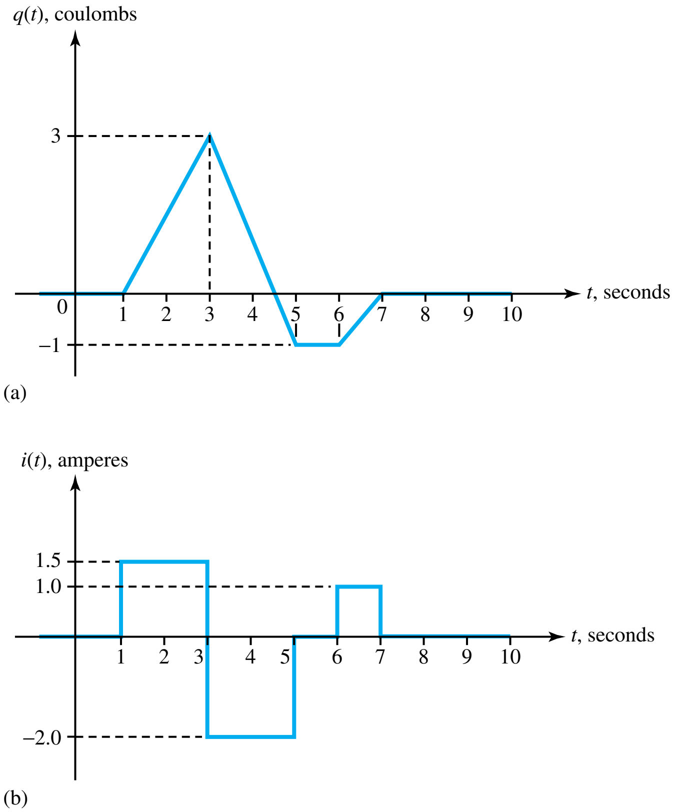


Figure E1.1.2 **(a)** Plot of $q(t)$. **(b)** Plot of $i(t)$.

Solution

(a) Applying Equation (1.1.5) and interpreting the first derivative as the slope, one obtains the plot shown in Figure E1.1.2(b).

(b) $I_{av} = (1/T)\int_0^T i\,dt$. Interpreting the integral as the area enclosed under the curve, one gets:

$$I_{av} = \frac{1}{(7-1)}[(1.5 \times 2) - (2.0 \times 2) + (0 \times 1) + (1 \times 1)] = 0$$

Note that the net charge transferred during the interval of 1 to 7 seconds is zero in this case.

EXAMPLE 1.1.3

Consider an infinitesimal length of 10^{-6} m of wire whose center is located at the point (1, 0, 0), carrying a current of 2 A in the positive direction of x.

(a) Find the magnetic flux density due to the current element at the point (0, 2, 2).

(b) Let another current element (of length 10^{-3} m) be located at the point (0, 2, 2), carrying a current of 1 A in the direction of $(-\bar{a}_y + \bar{a}_z)$. Evaluate the force on this current element due to the other element located at (1, 0, 0).

Solution

(a) $I_1 d\bar{l}_1 = 2 \times 10^{-6} \bar{a}_x$. The unit vector $\bar{a}_{12}$ is given by

$$\bar{a}_{12} = \frac{(0-1)\bar{a}_x + (2-0)\bar{a}_y + (2-0)\bar{a}_z}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{(-\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z)}{3}$$

Using the Biot–Savart law, Equation (1.1.9), one gets

$$[\bar{B}_1]_{(0,2,2)} = \frac{\mu_0}{4\pi} \frac{I_1\, d\bar{l}_1 \times \bar{a}_{12}}{R^2}$$

where μ_0 is the free-space permeability constant given in SI units as $4\pi \times 10^{-7}$ H/m, and R^2 in this case is $\{(0-1)^2 + (2-0)^2 + (2-0)^2\}$, or 9. Hence,

$$[\bar{B}_1]_{(0,2,2)} = \frac{4\pi \times 10^{-7}}{4\pi} \left[\frac{(2 \times 10^{-6}\bar{a}_x) \times (-\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z)}{9 \times 3} \right]$$
$$= \frac{10^{-7}}{27} \times 4 \times 10^{-6} (\bar{a}_z - \bar{a}_y)\ \mathrm{Wb/m^2}$$
$$= 0.15 \times 10^{-13} (\bar{a}_z - \bar{a}_y)\ \mathrm{T}$$

(b) $I_2\, d\bar{l}_2 = 10^{-3}(-\bar{a}_y + \bar{a}_z)$

$$d\bar{F}_{12} = I_2 d\bar{l}_2 \times \bar{B}_1$$
$$= \left[10^{-3}(-\bar{a}_y + \bar{a}_z)\right] \times \left[0.15 \times 10^{-13}(\bar{a}_z - \bar{a}_y)\right] = 0$$

Note that the force is zero since the current element $I_2\, d\bar{l}_2$ and the field $\bar{B}_1$ due to $I_1\, d\bar{l}_1$ at (0, 2, 2) are in the same direction.

The Biot–Savart law can be extended to find the magnetic flux density due to a current-carrying filamentary wire of any length and shape by dividing the wire into a number of infinitesimal elements and using superposition. The net force experienced by a current loop can be similarly evaluated by superposition.

Electric Potential and Voltage

When electrical forces act on a particle, it will possess potential *energy*. In order to describe the potential energy that a particle will have at a point x, the *electric potential* at point x is defined as

$$v(x) = \frac{dw(x)}{dq} \tag{1.1.13}$$

where $w(x)$ is the potential energy that a particle with charge q has when it is located at the position x. The zero point of potential energy can be chosen arbitrarily since only differences in energy have practical meaning. The point where electric potential is zero is known as the *reference point* or *ground point*, with respect to which potentials at other points are then described. The *potential difference* is known as the *voltage* expressed in volts (V) or joules per coulomb (J/C). If the potential at B is higher than that at A,

$$v_{BA} = v_B - v_A \tag{1.1.14}$$

which is positive. Obviously voltages can be either positive or negative numbers, and it follows that

$$v_{BA} = -v_{AB} \tag{1.1.15}$$

The voltage at point A, designated as v_A, is then the potential at point A with respect to the ground.

Energy and Power

If a charge dq gives up energy dw when going from point a to point b, then the voltage across those points is defined as

$$v = \frac{dw}{dq} \tag{1.1.16}$$

If dw/dq is positive, point a is at the higher potential. The voltage between two points is the work per unit positive charge required to move that charge between the two points. If dw and dq have the same sign, then energy is *delivered* by a positive charge going from a to b (or a negative charge going the other way). Conversely, charged particles *gain* energy inside a *source* where dw and dq have opposite polarities.

The *load* and *source* conventions are shown in Figure 1.1.3, in which point a is at a higher potential than point b. The load *receives* or *absorbs* energy because a positive charge goes in the direction of the current arrow from higher to lower potential. The source has a capacity to *supply* energy. The *voltage source* is sometimes known as an *electromotive force*, or *emf*, to convey the notation that it is a force that drives the current through the circuit.

The *instantaneous power* p is defined as the rate of doing work or the rate of change of energy dw/dt,

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi \tag{1.1.17}$$

The electric power consumed or produced by a circuit element is given by its voltage–current product, expressed in volt-amperes (VA) or watts (W). The energy over a time interval is found by integrating power,

$$w = \int_0^T p\,dt \tag{1.1.18}$$

which is expressed in watt-seconds or joules (J), or commonly in electric utility bills in kilowatt-hours (kWh). Note that 1 kWh equals 3.6×10^6 J.

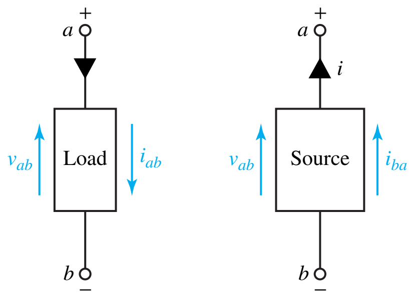


Figure 1.1.3 Load and source conventions.

EXAMPLE 1.1.4

A typical 12-V automobile battery, storing about 5 megajoules (MJ) of energy, is connected to a 4-A headlight system.

(a) Find the power delivered to the headlight system.

(b) Calculate the energy consumed in 1 hour of operation.

(c) Express the auto-battery capacity in ampere-hours (Ah) and compute how long the headlight system can be operated before the battery is completely discharged.

Solution

(a) Power delivered: $P = VI = 124 = 48\text{W}$.

(b) Assuming V and I remain constant, the energy consumed in 1 hour will equal

$$W = 48(60 \times 60) = 172.8 \times 10^3\text{J} = 172.8\text{kJ}$$

(c) 1 Ah $= (1\,\text{C/s})(3600\,\text{s}) = 3600\text{C}$. For the battery in question, $5 \times 10^6\text{J}/12\,\text{V} = 0.417 \times 10^6\text{C}$. Thus the auto-battery capacity is $0.417 \times 10^6/3600 \cong 116\,\text{Ah}$. Without completely discharging the battery, the headlight system can be operated for $116/4 = 29$ hours.

Sources and Loads

A source–load combination is represented in Figure 1.1.4. A *node* is a point at which two or more components or devices are connected together. A part of a circuit containing only one component, source, or device between two nodes is known as a *branch*. A voltage *rise* indicates an electric source, with the charge being raised to a higher potential, whereas a voltage *drop* indicates a load, with a charge going to a lower potential. The voltage *across* the source is the same as the voltage across the load in Figure 1.1.4. The current delivered by the source goes *through* the load. Ideally, with no losses, the power ($p = vi$) delivered by the source is consumed by the load.

When current flows out of the positive terminal of an electric source, it implies that non-electric energy has been transformed into electric energy. Examples include mechanical energy transformed into electric energy as in the case of a generator source, chemical energy changed into electric energy as in the case of a battery source, and solar energy converted into electric energy as in the case of a solar-cell source. On the other hand, when current flows in the direction of voltage drop, it implies that electric energy is transformed into nonelectric energy. Examples include electric energy converted into thermal energy as in the case of an electric heater, electric energy transformed into mechanical energy as in the case of motor load, and electric energy changed into chemical energy as in the case of a charging battery.

Batteries and ac outlets are the familiar electric sources. These are *voltage sources*. An *ideal voltage source* is one whose terminal voltage v is a specified function of time, regardless of the current i through the source. An ideal battery has a constant voltage V with respect to time, as shown in Figure 1.1.5(a). It is known as a dc source, because $i = I$ is a direct current. Figure 1.1.5(b) shows the symbol and time variation for a *sinusoidal voltage source* with $v = V_m \cos \omega t$. The positive sign on the source symbol indicates instantaneous polarity of the terminal at the higher potential whenever $\cos \omega t$ is positive. A sinusoidal source is generally termed an ac source because such a voltage source tends to produce an alternating current.

The concept of an *ideal current source*, although less familiar but useful as we shall see later, is defined as one whose current i is a specified function of time, regardless of the voltage across its terminals. The circuit symbols and the corresponding i–v curves for the ideal voltage and current sources are shown in Figure 1.1.6.

Even though ideal sources could theoretically produce infinite energy, one should recognize that infinite values are physically impossible. Various circuit laws and device representations or *models* are approximations of physical reality, and significant limitations of the idealized concepts or models need to be recognized. Simplified representations or models for physical devices are the most powerful tools in electrical engineering. As for ideal sources, the concept of constant V or constant I for dc sources and the general idea of v or i being a specified function of time should be understood.

When the source voltage or current is independent of all other voltages and currents, such sources are known as *independent sources*. There are *dependent* or *controlled sources*, whose

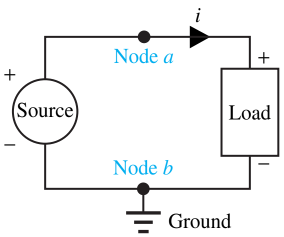


Figure 1.1.4 Source–load combination.

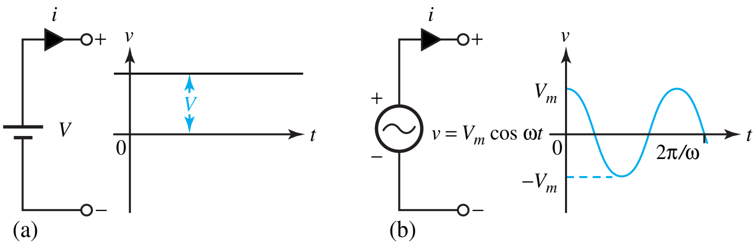


Figure 1.1.5 Voltage sources. **(a)** Ideal dc source (battery). **(b)** Ideal sinusoidal ac source.

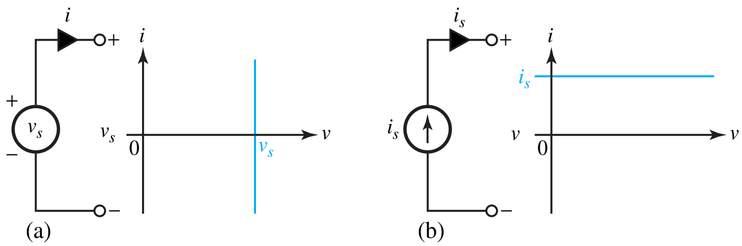


Figure 1.1.6 Circuit symbols and *i–v* curves. **(a)** Ideal voltage source. **(b)** Ideal current source.

voltage or current does depend on the value of some other voltage or current. As an example, a *voltage amplifier* producing an output voltage $v_{out} = Av_{in}$, where v_{in} is the input voltage and A is the constant-voltage amplification factor, is shown in Figure 1.1.7, along with its controlled-source model using the diamond-shaped symbol. Current sources controlled by a current or voltage will also be considered eventually.

Waveforms

We are often interested in *waveforms*, which may not be constant in time. Of particular interest is a *periodic waveform*, which is a *time-varying waveform* repeating itself over intervals of time $T > 0$.

$$f(t) = f(t \pm nT) \qquad n = 1,\ 2,\ 3, \cdots \tag{1.1.19}$$

The repetition time T of the waveform is called the *period* of the waveform. For a waveform to be periodic, it must continue indefinitely in time. The dc waveform of Figure 1.1.5(a) can be considered to be periodic with an infinite period. The *frequency* of a periodic waveform is the reciprocal of its period,

$$f = \frac{1}{T} \text{ Hertz (Hz)} \tag{1.1.20}$$

A sinusoidal or cosinusoidal waveform is typically described by

$$f(t) = A\ \sin(\omega t + \phi) \tag{1.1.21}$$

where A is the amplitude, ϕ is the phase offset, and $\omega = 2\pi f = 2\pi/T$ is the radian frequency of the wave. When $\phi = 0$, a sinusoidal wave results, and when $\phi = 90°$, a cosinusoidal wave results. The *average value* of a periodic waveform is the net positive area under the curve for one period, divided by the period,

$$F_{av} = \frac{1}{T} \int_0^T f(t)\, dt \tag{1.1.22}$$

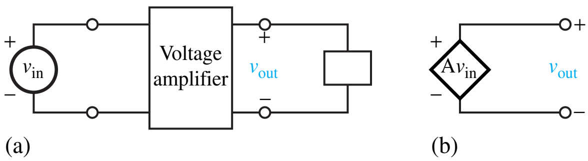


Figure 1.1.7 Voltage amplifier and its controlled-source model.

The *effective*, or *root-mean square* (rms), value is the square root of the average of $f^2(t)$,

$$F_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} \tag{1.1.23}$$

Determining the square of the function $f(t)$, then finding the mean (average) value, and finally taking the square root yields the rms value, known as effective value. This concept will be seen to be useful in comparing the effectiveness of different sources in delivering power to a resistor. The effective value of a periodic current, for example, is a constant, or dc value, which delivers the same average power to a resistor, as will be seen later.

For the special case of a dc waveform, the following holds:

$$f(t) = F; \qquad F_{\text{av}} = F_{\text{rms}} = F \tag{1.1.24}$$

For the sinusoid or cosinusoid, it can be seen that

$$f(t) = A\,\sin(\omega t + \phi); \qquad F_{\text{av}} = 0; \qquad F_{\text{rms}} = A/\sqrt{2} \cong 0.707\,\text{A} \tag{1.1.25}$$

The student is encouraged to show the preceding results using graphical and analytical means. Other common types of waveforms are *exponential* in nature,

$$f(t) = Ae^{-t/\tau} \tag{1.1.26a}$$

$$f(t) = A(1 - e^{-t/\tau}) \tag{1.1.26b}$$

where τ is known as the *time constant*. After a time of one time constant has elapsed, looking at Equation (1.1.26a), the value of the waveform will be reduced to 37% of its initial value; Equation (1.1.26b) shows that the value will rise to 63% of its final value. The student is encouraged to study the functions graphically and deduce the results.

EXAMPLE 1.1.5

A periodic current waveform in a rectifier is shown in Figure E1.1.5. The wave is sinusoidal for $\pi/3 \le \omega t \le \pi$, and is zero for the rest of the cycle. Calculate the rms and average values of the current.

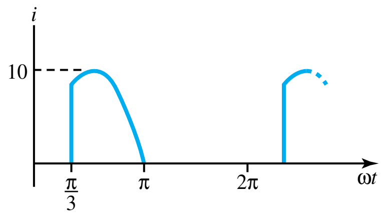


Figure E1.1.5

Solution

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi/3} i^2\,d(\omega t) + \int_{\pi/3}^{\pi} i^2\,d(\omega t) + \int_{\pi}^{2\pi} i^2\,d(\omega t)\right]}$$

Notice that ωt rather than t is chosen as the variable for convenience; $\omega = 2\pi f = 2\pi/T$; and integration is performed over three discrete intervals because of the discontinuous current function. Since $i = 0$ for $0 \le \omega t < \pi/3$ and $\pi \le \omega t \le 2\pi$,

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\pi/3}^{\pi} 10^2 \sin^2 \omega t \, d(\omega t)} = 4.49 \text{ A}$$

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\pi/3}^{\pi} 10 \sin \omega t \, d(\omega t) = 2.39 \text{ A}$$

Note that the base is the entire period 2π, even though the current is zero for a substantial part of the period.

1.2 LUMPED-CIRCUIT ELEMENTS

Electric *circuits* or *networks* are formed by interconnecting various devices, sources, and components. Although the effects of each element (such as heating effects, electric-field effects, or magnetic-field effects) are distributed throughout space, one often lumps them together as *lumped elements*. The *passive* components are the *resistance* R representing the heating effect, the *capacitance* C representing the electric-field effect, and the *inductance* L representing the magnetic-field effect. Their characteristics will be presented in this section. The capacitor models the relation between voltage and current due to changes in the accumulation of electric charge, and the inductor models the relation due to changes in magnetic flux linkages, as will be seen later. While these phenomena are generally distributed throughout an electric circuit, under certain conditions they can be considered to be concentrated at certain points and can therefore be represented by lumped parameters.

Resistance

An *ideal resistor* is a circuit element with the property that the current through it is linearly proportional to the potential difference across its terminals,

$$i = v/R = Gv, \text{ or } v = iR \tag{1.2.1}$$

which is known as *Ohm's law*, published in 1827. R is known as the resistance of the resistor with the SI unit of *ohms* (Ω), and G is the reciprocal of resistance called *conductance*, with the SI unit of *siemens* (S). The circuit symbols of fixed and variable resistors are shown in Figure 1.2.1, along with an illustration of Ohm's law. Most resistors used in practice are good approximations to *linear* resistors for large ranges of current, and their i–v characteristic (current versus voltage plot) is a straight line.

The value of resistance is determined mainly by the physical dimensions and the *resistivity* ρ of the material of which the resistor is composed. For a bar of resistive material of length l and cross-sectional area A the resistance is given by

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} \tag{1.2.2}$$

where ρ is the resistivity of the material in ohm-meters ($\Omega \cdot$ m), and σ is the *conductivity* of the material in S/m, which is the reciprocal of the resistivity. Metal wires are often considered as ideal

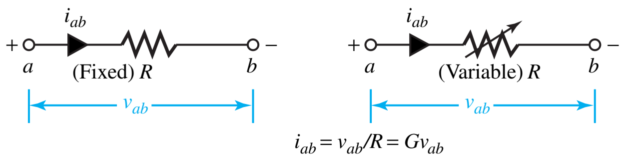


Figure 1.2.1 Circuit symbols of fixed and variable resistors and illustration of Ohm's law.

TABLE 1.2.1 Resistivity of Some Materials

Type	Material	$\rho(\Omega \cdot \mathrm{m})$
Conductors	Silver	16×10^{-9}
(at 20°C)	Copper	17×10^{-9}
	Gold	24×10^{-9}
	Aluminum	28×10^{-9}
	Tungsten	55×10^{-9}
	Brass	67×10^{-9}
	Sodium	0.04×10^{-6}
	Stainless steel	0.91×10^{-6}
	Iron	0.1×10^{-6}
	Nichrome	1×10^{-6}
	Carbon	35×10^{-6}
	Seawater	0.25
Semiconductors	Germanium	0.46
(at 27°C or 300 K)	Silicon	2.3×10^{3}
Insulators	Rubber	1×10^{12}
	Polystyrene	1×10^{15}

conductors with zero resistance as a good approximation. Table 1.2.1 lists values of ρ for some materials.

The resistivity of conductor metals varies linearly over normal operating temperatures according to

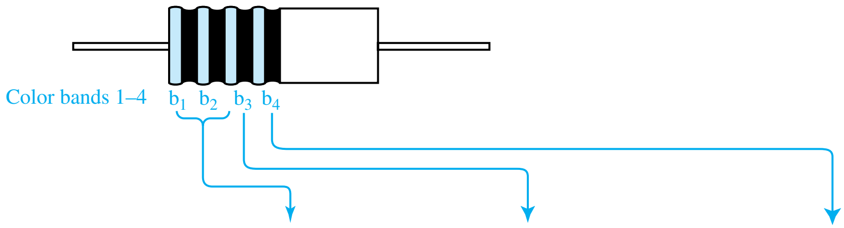
$$\rho_{T2} = \rho_{T1}\left(\frac{T_2 + T}{T_1 + T}\right) \tag{1.2.3}$$

where ρ_{T2} and ρ_{T1} are resistivities at temperatures T_2 and T_1, respectively, and T is a temperature constant that depends on the conductor material. All temperatures are in degrees Celsius. The conductor resistance also depends on other factors, such as spiraling, frequency (the skin effect which causes the ac resistance to be slightly higher than the dc resistance), and current magnitude in the case of magnetic conductors (e.g., steel conductors used for shield wires).

Practical resistors are manufactured in standard values, various resistance tolerances, several power ratings (as will be explained shortly), and in a number of different forms of construction. The three basic construction techniques are *composition* type, which uses carbon or graphite and is molded into a cylindrical shape, *wire-wound* type, in which a length of enamel-coated wire is wrapped around an insulating cylinder, and *metal-film* type, in which a thin layer of metal is vacuum deposited. Table 1.2.2 illustrates the standard color-coded bands used for evaluating resistance and their interpretation for the common carbon composition type. Sometimes a fifth band is also present to indicate reliability. Black is the least reliable color and orange is 1000 times more reliable than black.

For resistors ranging from 1 to 9.1 Ω, the standard resistance values are listed in Table 1.2.3. Other available values can be obtained by multiplying the values shown in Table 1.2.3 by factors

TABLE 1.2.2 Standard Color-Coded Bands for Evaluating Resistance and Their Interpretation



Color of Band	Digit of Band	Multiplier	% Tolerance in Actual Value
Black	0	10^0	—
Brown	1	10^1	—
Red	2	10^2	—
Orange	3	10^3	—
Yellow	4	10^4	—
Green	5	10^5	—
Blue	6	10^6	—
Violet	7	10^7	—
Grey	8	10^8	—
White	9	—	—
Gold	—	10^{-1}	± 5%
Silver	—	10^{-2}	± 10%
Black or no color	—	—	± 20%

Resistance value $= (10b_1 + b_2) \times 10^{b_3}\ \Omega$.

of 10 ranging from 10 Ω to about 22×10^6 Ω. For example, 8.2 Ω, 82 Ω, 820 Ω, . . . , 820 kΩ are standard available values.

The maximum allowable power dissipation or *power rating* is typically specified for commercial resistors. A common power rating for resistors used in electronic circuits is ¼ W; other ratings such as ⅛, ½, 1, and 2 W are available with composition-type resistors, whereas larger ratings are also available with other types. Variable resistors, known as *potentiometers*, with a movable contact are commonly found in rotary or linear form. Wire-wound potentiometers may have higher power ratings up to 1000 W.

The advent of integrated circuits has given rise to *packaged resistance arrays* fabricated by using film technology. These packages are better suited for automated manufacturing and are usually less costly than discrete resistors in large production runs.

An important property of the resistor is its ability to convert energy from electrical form into heat. The manufacturer generally states the maximum power dissipation of the resistor in watts. If more power than this is converted to heat by the resistor, the resistor will be damaged due to overheating. The instantaneous power absorbed by the resistor is given by

$$p(t) = v(t)i(t) = i^2 R = v^2/R = v^2 G \qquad (1.2.4)$$

where v is the voltage drop across the resistance and i is the current through the resistance. It can be shown (see Problem 1.2.13) that the average value of Equation (1.2.4) is given by

$$P_{av} = V_{rms} I_{rms} = I_{rms}^2 R = V_{rms}^2/R = V_{rms}^2 G \qquad (1.2.5)$$

for periodically varying current and voltage as a function of time. Equation (1.2.5) gives the expression for the power converted to heat by the resistor.

TABLE 1.2.3 Standard Available Values of Resistors

1.0	1.5	2.2	3.3	4.7	6.8
1.1	1.6	2.4	3.6	5.1	7.5
1.2	1.8	2.7	3.9	5.6	8.2
1.3	2.0	3.0	4.3	6.2	9.1

Series and *parallel* combinations of resistors occur very often. Figure 1.2.2 illustrates these combinations.

Figure 1.2.2(a) shows two resistors R_1 and R_2 in series sharing the voltage v in direct proportion to their values, while the same current i flows through both of them,

$$v = v_{AC} = v_{AB} + v_{BC} = iR_1 + iR_2 = i(R_1 + R_2) = iR_{eq}$$

or, when R_1 and R_2 are in series,

$$R_{eq} = R_1 + R_2 \tag{1.2.6}$$

Figure 1.2.2(b) shows two resistors in parallel sharing the current i in inverse proportion to their values, while the same voltage v is applied across each of them. At node B,

$$i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = v/\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \frac{v}{R_{eq}}$$

or, when R_1 and R_2 are in parallel,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad G_{eq} = G_1 + G_2 \tag{1.2.7}$$

Notice the *voltage division* shown in Figure 1.2.2(a), and the *current division* in Figure 1.2.2(b).

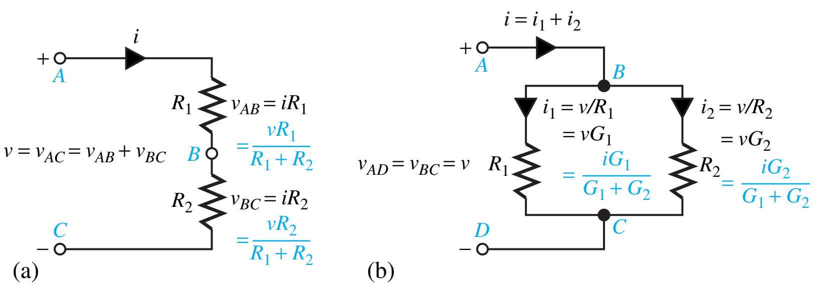


Figure 1.2.2 Resistances in series and in parallel. **(a)** R_1 and R_2 in series. **(b)** R_1 and R_2 in parallel.

EXAMPLE 1.2.1

A no. 14 gauge copper wire, commonly used in extension cords, has a circular wire diameter of 64.1 mils, where 1 mil = 0.001 inch.

(a) Determine the resistance of a 100-ft-long wire at 20°C.

(b) If such a 2-wire system is connected to a 110-V (rms) residential source outlet in order to power a household appliance drawing a current of 1 A (rms), find the rms voltage at the load terminals.

(c) Compute the power dissipated due to the extension cord.

(d) Repeat part (a) at 50°C, given that the temperature constant for copper is 241.5°C.

Solution

(a) $d = 64.1$ mils $= 64.1 \times 10^{-3}$ in $= 64.1 \times 10^{-3} \times 2.54$ cm/1 in $\times$ 1 m/100 cm $= 1.628 \times 10^{-3}$ m. From Table 1.2.1, ρ of copper at 20°C is 17×10^{-9} m,

$$l = 100 \text{ ft} = 100 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 30.48 \text{ m}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (1.628 \times 10^{-3})^2}{4} = 2.08 \times 10^{-6} \text{ m}^2$$

Per Equation (1.2.2),

$$R_{20^\circ\mathrm{C}} = \frac{17 \times 10^{-9} \times 30.48}{2.08 \times 10^{-6}} \cong 0.25\ \Omega$$

(b) Rms voltage at load terminals, V = 110 − (0.25)2 = 109.5 V (rms). Note that two 100-ft-long wires are needed for the power to be supplied.

(c) Power dissipated, per Equation (1.2.5), $P = (1)^2(0.25)(2) = 0.5$ W.

(d) Per Equation (1.2.3),

$$\rho_{50^\circ\mathrm{C}} = \rho_{20^\circ\mathrm{C}} \left(\frac{50 + 241.5}{20 + 241.5} \right) = \frac{17 \times 10^{-9} \times 291.5}{261.5} = 18.95 \times 10^{-9}\ \Omega \cdot \text{m}$$

Hence,

$$R_{50^\circ\mathrm{C}} = \frac{18.95 \times 10^{-9} \times 30.48}{2.08 \times 10^{-6}} \cong 0.28\ \Omega$$

EXAMPLE 1.2.2

(a) Consider a series–parallel combination of resistors as shown in Figure E1.2.2(a). Find the equivalent resistance as seen from terminals A–B.

(b) Determine the current I and power P delivered by a 10-V dc voltage source applied at terminals A–B, with A being at higher potential than B.

(c) Replace the voltage source by an equivalent current source at terminals A–B.

(d) Show the current and voltage distribution clearly in all branches of the original circuit configuration.

Solution

(a) The circuit is reduced as illustrated in Figure E1.2.2(b).

(b) $I = 5$ A; $P = VI = I^2R = V^2/R = 50$ W [see Figure E1.2.2(c)].

(c) See Figure E1.2.2(d).

(d) See Figure E1.2.2(e).

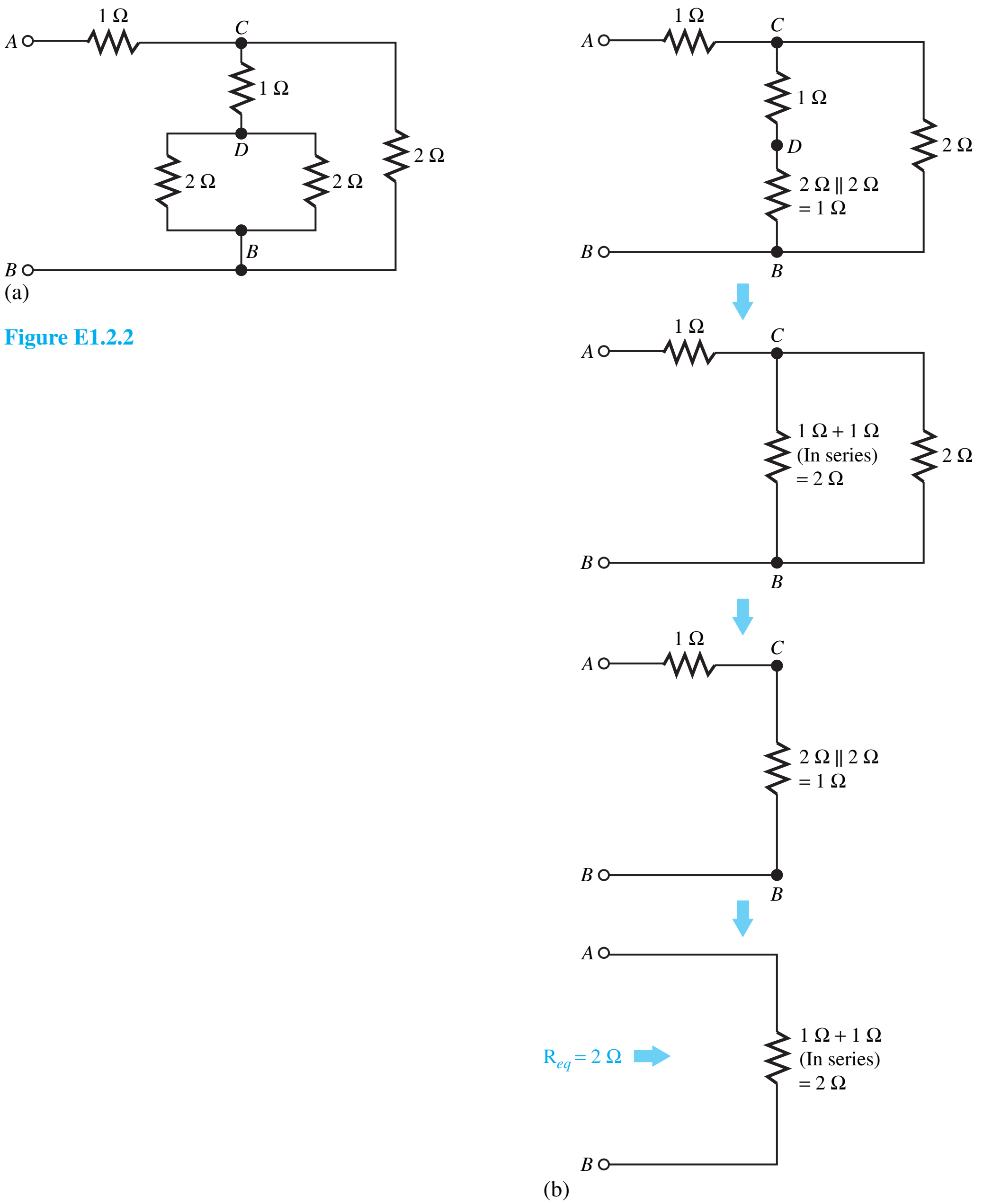
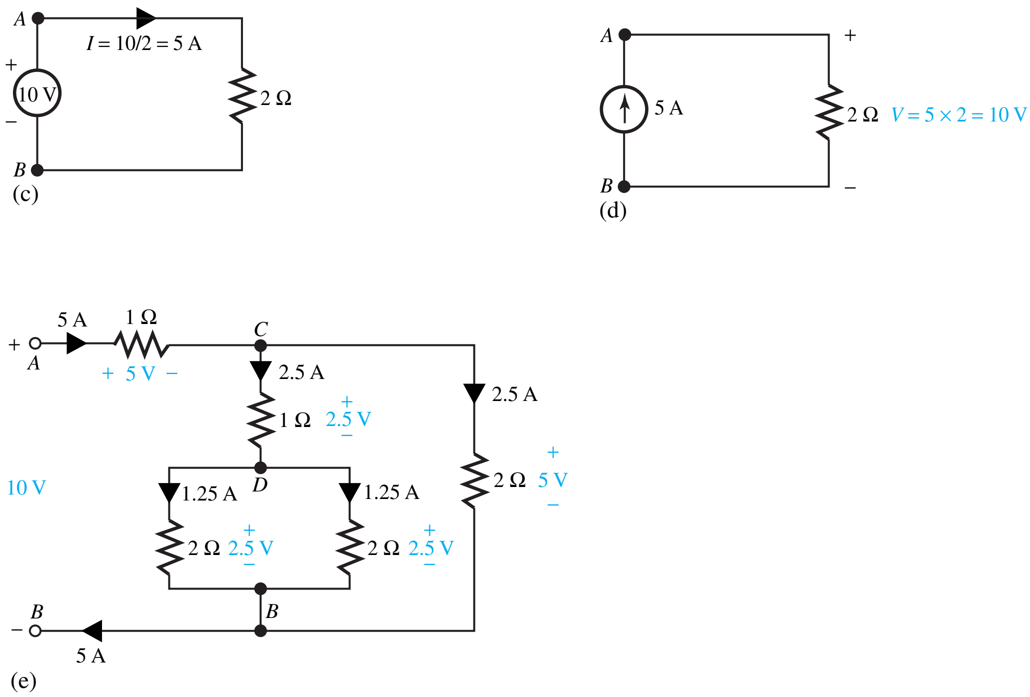


Figure E1.2.2



Maximum Power Transfer

In order to investigate the power transfer between a practical source and a load connected to it, let us consider Figure 1.2.3, in which a constant voltage source v with a known internal resistance R_S is connected to a variable load resistance R_L. Note that when R_L is equal to zero, it is called a *short circuit*, in which case v_L becomes zero and i_L is equal to v/R_S. When R_L approaches infinity, it is called an *open circuit*, in which case i_L becomes zero and v_L is equal to v. One is generally interested to find the value of the load resistance that will absorb maximum power from the source.

The power P_L absorbed by the load is given by

$$P_L = i_L^2 R_L \tag{1.2.8}$$

where the load current i_L is given by

$$i_L = \frac{v^2}{R_S + R_L} \tag{1.2.9}$$

Substituting Equation (1.2.9) in Equation (1.2.8), one gets

$$P_L = \frac{v^2}{(R_S + R_L)^2} R_L \tag{1.2.10}$$

For given fixed values of v and R_S, in order to find the value of R_L that maximizes the power absorbed by the load, one sets the first derivative dP_L/dR_L equal to zero,

$$\frac{dP_L}{dR_L} = \frac{v^2(R_L + R_S)^2 - 2v^2 R_L (R_L + R_S)}{(R_L + R_S)^4} = 0 \tag{1.2.11}$$

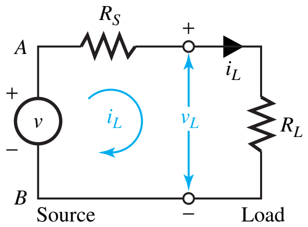


Figure 1.2.3 Power transfer between source and load. *Note:* $R_L = 0$ implies short circuit; $v_L = 0$ and $i_L = \frac{v}{R_S}$ and $R_L \to \bullet$ implies open circuit; $i_L = 0$ and $v_L = v$.

which leads to the following equation:

$$(R_L + R_S)^2 - 2R_L(R_L + R_S) = 0 \tag{1.2.12}$$

The solution of Equation (1.2.12) is given by

$$R_L = R_S \tag{1.2.13}$$

That is to say, in order to transfer maximum power to a load, the load resistance must be *matched* to the source resistance or, in other words, they should be equal to each other.

A problem related to power transfer is that of *source loading*. Figure 1.2.4(a) illustrates a practical voltage source (i.e., an ideal voltage source along with a series internal source resistance) connected to a load resistance; Figure 1.2.4(b) shows a practical current source (i.e., an ideal current source along with a parallel or shunt internal source resistance) connected to a load resistance. It follows from Figure 1.2.4(a) that

$$v_L = v - v_{\text{int}} = v - i_L R_S \tag{1.2.14}$$

where v_{int} is the internal voltage drop within the source, which depends on the amount of current drawn by the load. As seen from Equation (1.2.14), the voltage actually seen by the load v_L is somewhat lower than the *open-circuit voltage* of the source. When the load resistance R_L is infinitely large, the load current i_L goes to zero, and the load voltage v_L is then equal to the open-circuit voltage of the source v. Hence, it is desirable to have as small an internal resistance as possible in a practical voltage source.

From Figure 1.2.4(b) it follows that

$$i_L = i - i_{\text{int}} = i - \frac{v_L}{R_S} \tag{1.2.15}$$

where i_{int} is the internal current drawn away from the load because of the presence of the internal source resistance. Thus the load will receive only part of the *short-circuit current* available from the source. When the load resistance R_L is zero, the load voltage v_L goes to zero, and the load

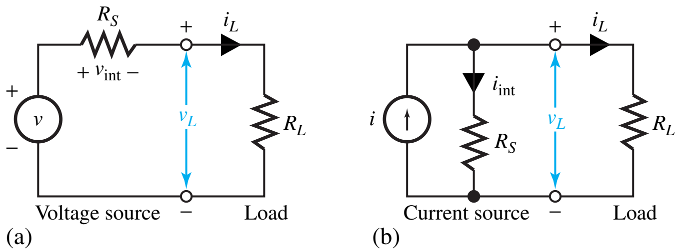


Figure 1.2.4 Source-loading effects.

current i_L is then equal to the short-circuit current of the source i. Hence, it is desirable to have as large an internal resistance as possible in a practical current source.

Capacitance

An *ideal capacitor* is an energy-storage circuit element (with no loss associated with it) representing the electric-field effect. The capacitance in farads (F) is defined by

$$C = q/v \tag{1.2.16}$$

where q is the charge on each conductor, and v is the potential difference between the two perfect conductors. With v being proportional to q, C is a constant determined by the geometric configuration of the two conductors. Figure 1.2.5(a) illustrates a two-conductor system carrying $+q$ and $-q$ charges, respectively, forming a capacitor.

The general circuit symbol for a capacitor is shown in Figure 1.2.5(b), where the current entering one terminal of the capacitor is equal to the rate of buildup of charge on the plate attached to that terminal,

$$i(t) = \frac{dq}{dt} = C\,\frac{dv}{dt} \tag{1.2.17}$$

in which C is assumed to be a constant and not a function of time (which it could be, if the separation distance between the plates changed with time).

The terminal v–i relationship of a capacitor can be obtained by integrating both sides of Equation (1.2.17),

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau \tag{1.2.18}$$

which may be rewritten as

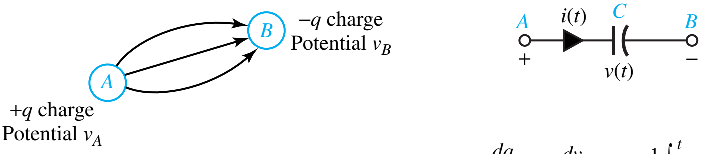
$$v(t) = \frac{1}{C}\int_{0}^{t} i(\tau)\,d\tau + \frac{1}{C}\int_{-\infty}^{0} i(\tau)\,d\tau = \frac{1}{C}\int_{0}^{t} i(\tau)\,d\tau + v(0) \tag{1.2.19}$$

where $v(0)$ is the *initial* capacitor voltage at $t = 0$.

The instantaneous power delivered to the capacitor is given by

$$p(t) = v(t)i(t) = C\,v(t)\,\frac{dv(t)}{dt} \tag{1.2.20}$$

whose average value can be shown (see Problem 1.2.13) to be zero for sinusoidally varying current and voltage as a function of time. The energy stored in a capacitor at a particular time is found by integrating,



Potential difference $= v = v_A - v_B$; $C = q/v$

(a)

$i(t) = \frac{dq}{dt} = C\frac{dv}{dt}$; $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = \frac{1}{C}\int_{0}^{t} i(\tau)\,d\tau + v(0)$

(b)

Figure 1.2.5 Capacitor. **(a)** Two perfect conductors carrying $+q$ and $-q$ charges. **(b)** Circuit symbol.

$$w(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = C \int_{-\infty}^{t} v(\tau)\frac{dv(\tau)}{d\tau} = \frac{1}{2}Cv^2(t) - \frac{1}{2}Cv^2(-\infty) \tag{1.2.21}$$

Assuming the capacitor voltage to be zero at $t = -\infty$, the stored energy in the capacitor at some time t is given by

$$w(t) = \frac{1}{2}Cv^2(t) \tag{1.2.22}$$

which depends only on the voltage of the capacitor at that time, and represents the stored energy in the electric field between the plates due to the separation of charges.

If the voltage across the capacitor does not change with time, no current flows, as seen from Equation (1.2.17). Thus the capacitor acts like an open circuit, and the following relations hold:

$$C = \frac{Q}{V}; \quad I = 0, \quad W = \frac{1}{2}CV^2 \tag{1.2.23}$$

An ideal capacitor, once charged and disconnected, the current being zero, will retain a potential difference for an indefinite length of time. Also, the voltage across a capacitor cannot change value instantaneously, while an instantaneous change in the capacitor current is quite possible. The student is encouraged to reason through and justify the statement made here by recalling Equation (1.2.17).

Series and parallel combinations of capacitors are often encountered. Figure 1.2.6 illustrates these.

It follows from Figure 1.2.6(a),

$$v = v_{\mathrm{AC}} = v_{\mathrm{AB}} + v_{\mathrm{BC}}$$

$$\frac{dv}{dt} = \frac{dv_{\mathrm{AB}}}{dt} + \frac{dv_{\mathrm{BC}}}{dt} = \frac{i}{C_1} + \frac{i}{C_2} = i\left(\frac{C_1 + C_2}{C_1C_2}\right) = \frac{i}{C_{\mathrm{eq}}}$$

or, when C_1 and C_2 are in series,

$$C_{\mathrm{eq}} = \frac{C_1C_2}{C_1 + C_2} \quad \text{or} \quad \frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{1.2.24}$$

Referring to Figure 1.2.6(b), one gets

$$i = i_1 + i_2 = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} = (C_1 + C_2)\frac{dv}{dt} = C_{\mathrm{eq}}\frac{dv}{dt}$$

or, when C_1 and C_2 are in parallel,

$$C_{\mathrm{eq}} = C_1 + C_2 \tag{1.2.25}$$

Note that capacitors in parallel combine as resistors in series, and capacitors in series combine as resistors in parallel.

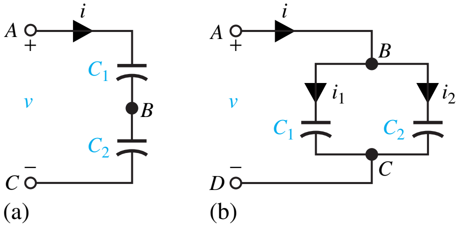


Figure 1.2.6 Capacitors in series and in parallel. **(a)** C_1 and C_2 in series. **(b)** C_1 and C_2 in parallel.

The *working voltage* for a capacitor is generally specified by the manufacturer, thereby giving the maximum voltage that can safely be applied between the capacitor terminals. Exceeding this limit may result in the breakdown of the insulation and then the formation of an electric arc between the capacitor plates. Unintentional or *parasitic* capacitances that occur due to the proximity of circuit elements may have serious effects on the circuit behavior.

Physical capacitors are often made of tightly rolled sheets of metal film, with a dielectric (paper or nylon) sandwiched in between, in order to increase their capacitance values (or ability to store energy) for a given size. Table 1.2.4 lists the range of general-purpose capacitances together with the maximum voltages and frequencies for different types of dielectric materials. Practical capacitors come in a wide range of values, shapes, sizes, voltage ratings, and constructions. Both fixed and adjustable devices are available. Larger capacitors are of the electrolytic type, using aluminum oxide as the dielectric.

TABLE 1.2.4 Characteristics of General-Purpose Capacitors

Material	Capacitance Range	Maximum Voltage Range (V)	Frequency Range (Hz)
Mica	1 pF to 0.1 μF	50–600	10^3–10^{10}
Ceramic	10 pF to 1 μF	50–1600	10^3–10^{10}
Mylar	0.001 F to 10 μF	50–600	10^2–10^8
Paper	10 pF to 50 μF	50–400	10^2–10^8
Electrolytic	0.1 μF to 0.2 F	3–600	10–10^4

Note: 1 pF $= 10^{-12}$ F; 1 μF $= 10^{-6}$ F.

EXAMPLE 1.2.3

(a) Consider a 5-μF capacitor to which a voltage $v(t)$ is applied, shown in Figure E1.2.3(a), top. Sketch the capacitor current and stored energy as a function of time.

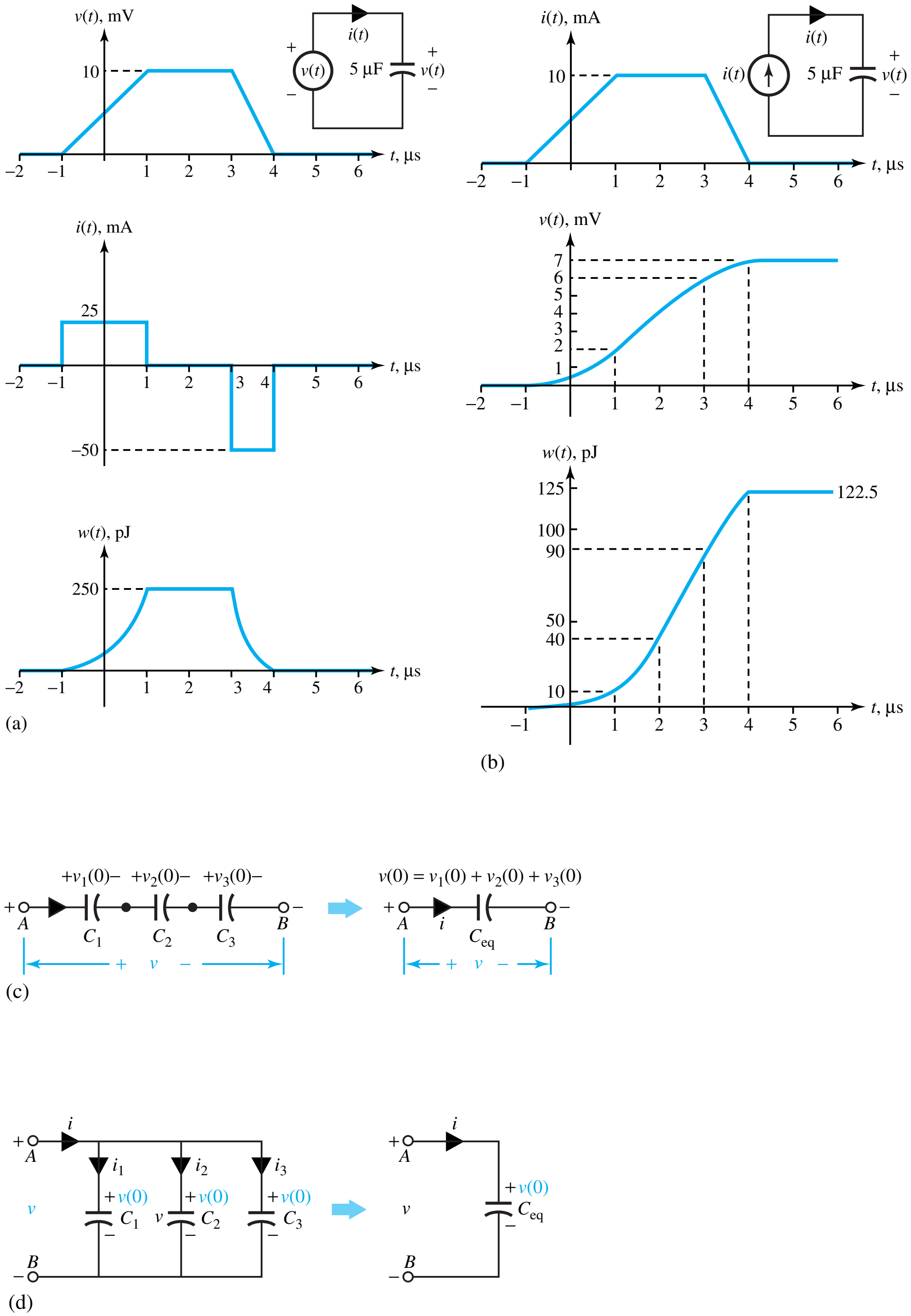
(b) Let a current source $i(t)$ be attached to the 5-μF capacitor instead of the voltage source of part (a), shown in Figure E1.2.3(b), top. Sketch the capacitor voltage and energy stored as a function of time.

(c) If three identical 5-μF capacitors with an initial voltage of 1 mV are connected (i) in series and (ii) in parallel, find the equivalent capacitances for both cases.

Solution

(a) From Figure E1.2.3(a) it follows that

$$
\begin{aligned}
v(t) &= 0\,, && t \le -1\ \mu\text{s} \\
&= 5(t+1)\ \text{mV}, && -1 \le t \le 1\ \mu\text{s} \\
&= 10\ \text{mV}, && 1 \le t \le 3\ \mu\text{s} \\
&= -10(t-4)\ \text{mV}, && 3 \le t \le 4\ \mu\text{s} \\
&= 0\,, && 4 \le t\ \mu\text{s}
\end{aligned}
$$



v(t), mV
i(t)
5 μF
t, μs
i(t), mA
w(t), pJ
(a)
(b)
122.5
+v_1(0)− +v_2(0)− +v_3(0)−
v(0) = v_1(0) + v_2(0) + v_3(0)
C_1 C_2 C_3 C_eq
(c)
(d)

Figure E1.2.3

Since

$$i(t) = C\frac{dv}{dt} = (5 \times 10^{-6})\frac{dv}{dt}$$

it follows that

$$\begin{aligned} i(t) &= 0, & t &\le -1\ \mu\text{s} \\ &= 25\ \text{mA}, & -1 \le t &\le 1\ \mu\text{s} \\ &= 0, & 1 \le t &\le 3\ \mu\text{s} \\ &= -50\ \text{mA}, & 3 \le t &\le 4\ \mu\text{s} \\ &= 0, & 4 &\le t\ \mu\text{s} \end{aligned}$$

which is sketched in the center of Figure E1.2.3(a).

Since the energy stored at any instant is

$$w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(5 \times 10^{-6})v^2(t)$$

it follows that:

$$\begin{aligned} w(t) &= 0, & t &\le -1\ \mu\text{s} \\ &= 62.5\,(t^2 + 2t + 1)\ \text{pJ}, & -1 \le t &\le 1\ \mu\text{s} \\ &= 250\ \text{pJ}, & 1 \le t &\le 3\ \mu\text{s} \\ &= 250\,(t^2 - 8t + 16)\ \text{pJ}, & 3 \le t &\le 4\ \mu\text{s} \\ &= 0, & 4 &\le t\ \mu\text{s} \end{aligned}$$

which is sketched at the bottom of Figure E1.2.3(a).

(b) From Figure E1.2.3(b) it follows that

$$\begin{aligned} i(t) &= 0, & t &\le -1\ \mu\text{s} \\ &= 5\,(t + 1)\ \text{mA}, & -1 \le t &\le 1\ \mu\text{s} \\ &= 10\ \text{mA}, & 1 \le t &\le 3\ \mu\text{s} \\ &= -10\,(t - 4)\ \text{mA}, & 3 \le t &\le 4\ \mu\text{s} \\ &= 0, & 4 &\le t\ \mu\text{s} \end{aligned}$$

Since

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = \frac{1}{5 \times 10^{-6}}\int_{-\infty}^{t} i(\tau)\,d\tau$$

it follows that

$$\begin{aligned} v(t) &= 0, & t &\le -1\ \mu\text{s} \\ &= \left(\frac{t^2}{2} + t + \frac{1}{2}\right)\ \text{mV}, & -1 \le t &\le 1\ \mu\text{s} \end{aligned}$$

$$= 2t \text{ mV}, \qquad 1 \le t \le 3\ \mu\text{s}$$
$$= -t^2 + 8t - 9 \text{ mV}, \qquad 3 \le t \le 4\ \mu\text{s}$$
$$= 7 \text{ mV}, \qquad 4 \le t\ \mu\text{s}$$

which is sketched in the center of Figure E1.2.3(b).

Since the energy stored at any instant is

$$w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(5 \times 10^{-6})v^2(t)$$

it follows that

$$w(t) = 0, \qquad t \le -1\ \mu\text{s}$$
$$= 2.5\,\left(\frac{t^2}{2} + t + \frac{1}{2}\right)^2 \text{ pJ}, \qquad -1 \le t \le 1\ \mu\text{s}$$
$$= 10t^2 \text{ pJ}, \qquad 1 \le t \le 3\ \mu\text{s}$$
$$= 2.5\,(-t^2 + 8t - 9)^2 \text{ pJ}, \qquad 3 \le t \le 4\ \mu\text{s}$$
$$= 122.5\text{pJ}, \qquad 4 \le t\ \mu\text{s}$$

which is sketched at the bottom of Figure E1.2.3(b).

(c) (i)

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{3}{5 \times 10^{-6}}, \qquad \text{or} \qquad C_{\text{eq}} = \frac{5}{3} \times 10^{-6}\text{F} = \frac{5}{3}\ \mu\text{F},$$

with an initial voltage $v(0) = 3$ mV [Figure E1.2.3(c)].

(ii)

$$C_{\text{eq}} = C_1 + C_2 + C_3 = 3 \times 5 \times 10^{-6} \text{ F} = 15\ \mu\text{F}$$

with an initial voltage $v(0) = 1$ mV [Figure E1.2.3(d)].

Inductance

An *ideal inductor* is also an energy-storage circuit element (with no loss associated with it) like a capacitor, but representing the magnetic-field effect. The inductance in henrys (H) is defined by

$$L = \frac{\lambda}{i} = \frac{N\psi}{i} \tag{1.2.26}$$

where λ is the magnetic-flux linkage in weber-turns (Wb·t), N is the number of turns of the coil, and $N\psi$ is the magnetic flux in webers (Wb) produced by the current i in amperes (A). Figure 1.2.7(a) illustrates a single inductive coil or an inductor of N turns carrying a current i that is linked by its own flux.

The general circuit symbol for an inductor is shown in Figure 1.2.7(b). According to Faraday's law of induction, one can write

$$v(t) = \frac{d\lambda}{dt} = \frac{d(N\psi)}{dt} = N\frac{d\psi}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} \tag{1.2.27}$$

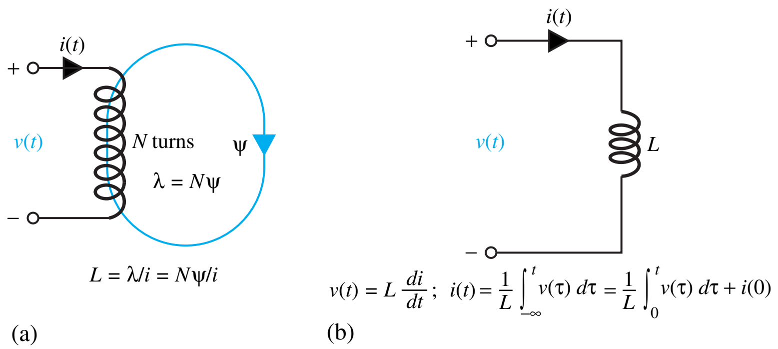


Figure 1.2.7 An inductor. **(a)** A single inductive coil of N turns. **(b)** Circuit symbol.

where L is assumed to be a constant and not a function of time (which it could be if the physical shape of the coil changed with time). Mathematically, by looking at Equations (1.2.17) and (1.2.27), the inductor is the *dual* of the capacitor. That is to say, the terminal relationship for one circuit element can be obtained from that of the other by interchanging v and i, and also by interchanging L and C.

The terminal i–v relationship of an inductor can be obtained by integrating both sides of Equation (1.2.27),

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau \tag{1.2.28}$$

which may be rewritten as

$$i(t) = \frac{1}{L}\int_{0}^{t} v(\tau)\,d\tau + \frac{1}{L}\int_{-\infty}^{0} v(\tau)\,d\tau = \frac{1}{L}\int_{0}^{t} v(\tau)\,d\tau + i(0) \tag{1.2.29}$$

where $i(0)$ is the initial inductor current at $t = 0$.

The instantaneous power delivered to the inductor is given by

$$p(t) = v(t)i(t) = Li(t)\frac{di(t)}{dt} \tag{1.2.30}$$

whose average value can be shown (see Problem 1.2.13) to be zero for sinusoidally varying current and voltage as a function of time. The energy stored in an inductor at a particular time is found by integrating,

$$w(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty) \tag{1.2.31}$$

Assuming the inductor current to be zero at $t = -\infty$, the stored energy in the inductor at some time t is given by

$$w(t) = \frac{1}{2}Li^2(t) \tag{1.2.32}$$

which depends only on the inductor current at that time, and represents the stored energy in the magnetic field produced by the current carried by the coil.

If the current flowing through the coil does not change with time, no voltage across the coil exists, as seen from Equation (1.2.27). The following relations hold:

$$L = \frac{\lambda}{I}; \qquad V = 0; \qquad W = \frac{1}{2}LI^2 \tag{1.2.33}$$

Under dc conditions, an ideal inductor acts like an ideal wire, or short circuit. Note that the current through an inductor cannot change value instantaneously. However, there is no reason to rule out an instantaneous change in the value of the inductor voltage. The student should justify the statements made here by recalling Equation (1.2.27).

If the medium in the flux path has a linear magnetic characteristic (i.e., constant permeability), then the relationship between the flux linkages λ and the current i is *linear*, and the slope of the linear λ–i characteristic gives the *self-inductance*, defined as flux linkage per ampere by Equation (1.2.26). While the inductance in general is a function of the geometry and permeability of the material medium, in a linear system it is independent of voltage, current, and frequency. If the inductor coil is wound around a ferrous core such as iron, the λ–i relationship will be *nonlinear* and even multivalued because of hysteresis. In such a case the inductance becomes a function of the current, and the inductor is said to be nonlinear. However, we shall consider only linear inductors here.

Series and parallel combinations of inductors are often encountered. Figure 1.2.8 illustrates these.

By invoking the principle of *duality*, it can be seen that the inductors in series combine like resistors in series and capacitors in parallel; the inductors in parallel combine like resistors in parallel and capacitors in series. Thus, when L_1 and L_2 are in series,

$$L_{eq} = L_1 + L_2 \tag{1.2.34}$$

and when L_1 and L_2 are in parallel,

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \tag{1.2.35}$$

A practical inductor may have considerable resistance in the wire of a coil, and sizable capacitances may exist between various turns. A possible *model* for a practical inductor could be a combination of ideal elements: a combination of resistance and inductance in series, with a capacitance in parallel. Techniques for modeling real circuit elements will be used extensively in later chapters.

Practical inductors range from about 0.1 μH to hundreds of millihenrys. Some, meant for special applications in power supplies, can have values as large as several henrys. In general, the larger the inductance, the lower its frequency is in its usage. The smallest inductance values are generally used at radio frequencies. Although inductors have many applications, the total demand does not even remotely approach the consumption of resistors and capacitors. Inductors generally tend to be rather bulky and expensive, especially in low-frequency applications. Industry-wide standardization for inductors is not done to the same degree as for more frequently used devices such as resistors and capacitors.

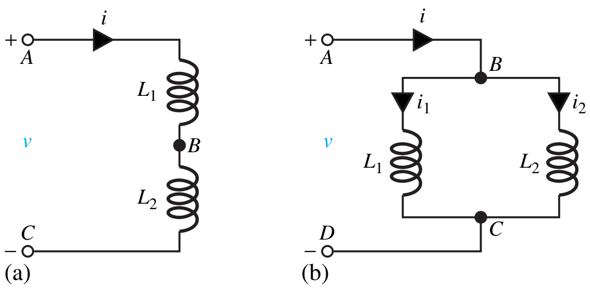


Figure 1.2.8 Inductors in series and parallel. **(a)** L_1 and L_2 in series. **(b)** L_1 and L_2 in parallel.

EXAMPLE 1.2.4

(a) Consider a 5-μH inductor to which a current source $i(t)$ is attached, as shown in Figure E1.2.3(b). Sketch the inductor voltage and stored energy as a function of time.

(b) Let a voltage source $v(t)$ shown in Figure E1.2.3(a) be applied to the 5-μH inductor instead of the current source in part (a). Sketch the inductor current and energy stored as a function of time.

(c) If three identical 5-μH inductors with initial current of 1 mA are connected (i) in series and (ii) in parallel, find the equivalent inductance for each case.

Solution

(a) From the principle of duality and for the given values, it follows that the inductor-voltage waveform is the same as the capacitor-current waveform of Example 1.2.3(a), in which $i(t)$ is to be replaced by $v(t)$, and 25 and -50 mA are to be replaced by 25 and -50 mV. The stored energy $w(t)$ is the same as in the solution of Example 1.2.3(a).

(b) The solution is the same as that of Example 1.2.3(b), except that $v(t)$ in mV is to be replaced by $i(t)$ in mA.

(c) (i) Looking at the solution of Example 1.2.3(c), part (ii),

$$L_{\text{eq}} = L_1 + L_2 + L_3 = 3 \times 5 \times 10^{-6}\text{ H} = 15\ \mu\text{H}$$

with an initial current $i(0) = 1$ mA.

(ii) Following the solution of Example 1.2.3(c), part (i),

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{3}{5 \times 10^{-6}} \qquad \textit{or} \qquad L_{\text{eq}} = \frac{5}{3} \times 10^{-6}\text{ H} = \frac{5}{3}\ \mu\text{H}$$

with an initial current $i(0) = 3$ mA.

When more than one loop or circuit is present, the flux produced by the current in one loop may link another loop, thereby inducing a current in that loop. Such loops are said to be mutually coupled, and there exists a *mutual inductance* between such loops. The mutual inductance between two circuits is defined as the flux linkage produced in one circuit by a current of 1 ampere in the other circuit. Let us now consider a pair of mutually coupled inductors, as shown in Figure 1.2.9. The self-inductances L_{11} and L_{22} of inductors 1 and 2, respectively, are given by

$$L_{11} = \frac{\lambda_{11}}{i_1} \tag{1.2.36}$$

and

$$L_{22} = \frac{\lambda_{22}}{i_2} \tag{1.2.37}$$

where λ_{11} is the flux linkage of inductor 1 produced by its own current i_1, and λ_{22} is the flux linkage of inductor 2 produced by its own current i_2. The mutual inductances L_{12} and L_{21} are given by

$$L_{12} = \frac{\lambda_{12}}{i_2} \tag{1.2.38}$$

and

$$L_{21} = \frac{\lambda_{21}}{i_1} \tag{1.2.39}$$

where λ_{12} is the flux linkage of inductor 1 produced by the current i_2 in inductor 2, and λ_{21} is the flux linkage of inductor 2 produced by the current i_1 in inductor 1.

If a current of i_1 flows in inductor 1 while the current in inductor 2 is zero, the equivalent fluxes are given by

$$\psi_{11} = \frac{\lambda_{11}}{N_1} \tag{1.2.40}$$

and

$$\psi_{21} = \frac{\lambda_{21}}{N_2} \tag{1.2.41}$$

where N_1 and N_2 are the number of turns of inductors 1 and 2, respectively. That part of the flux of inductor 1 that does not link any turn of inductor 2 is known as the equivalent *leakage flux* of inductor 1,

$$\psi_{l1} = \psi_{11} - \psi_{21} \tag{1.2.42}$$

Similarly,

$$\psi_{l2} = \psi_{22} - \psi_{12} \tag{1.2.43}$$

The *coefficient of coupling* is given by

$$k = \sqrt{k_1 k_2} \tag{1.2.44}$$

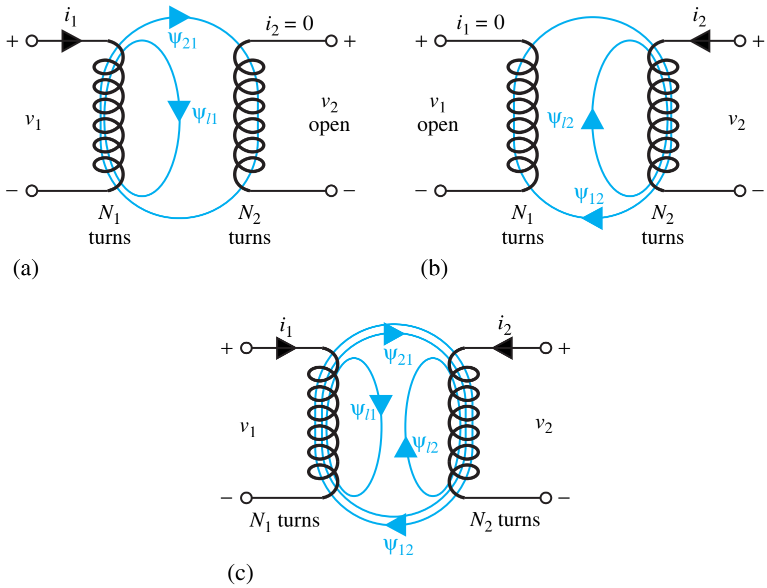


Figure 1.2.9 Mutually coupled inductors.

where $k_1 = \psi_{21}/\psi_{11}$ and $k_2 = \psi_{12}/\psi_{22}$. When k approaches unity, the two inductors are said to be tightly coupled; and when k is much less than unity, they are said to be loosely coupled. While the coefficient of coupling can never exceed unity, it may be as high as 0.998 for iron-core transformers; it may be smaller than 0.5 for air-core transformers.

When there are only two inductively coupled circuits, the symbol M is frequently used to represent the mutual inductance. It can be shown that the mutual inductance between two electric circuits coupled by a homogeneous medium of constant permeability is reciprocal,

$$M = L_{12} = L_{21} = k\sqrt{L_{11}L_{22}} \tag{1.2.45}$$

The energy considerations that lead to such a conclusion are taken up in Problem 1.2.30 as an exercise for the student.

Let us next consider the energy stored in a pair of mutually coupled inductors,

$$W_m = \frac{i_1\lambda_1}{2} + \frac{i_2\lambda_2}{2} \tag{1.2.46}$$

where λ_1 and λ_2 are the total flux linkages of inductors 1 and 2, respectively, and subscript m denotes association with the magnetic field. Equation (1.2.46) may be rewritten as

$$\begin{aligned} W_m &= \frac{i_1}{2}(\lambda_{11} + \lambda_{12}) + \frac{i_2}{2}(\lambda_{22} + \lambda_{21}) \\ &= \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2 + \frac{1}{2}L_{21}i_1i_2 \end{aligned}$$

or

$$W_m = \frac{1}{2}L_{11}i_1^2 + Mi_1i_2 + \frac{1}{2}L_{22}i_2^2 \tag{1.2.47}$$

Equation (1.2.47) is valid whether the inductances are constant or variable, so long as the magnetic field is confined to a uniform medium of constant permeability.

Where there are n coupled circuits, the energy stored in the magnetic field can be expressed as

$$W_m = \sum_{j=1}^{n}\sum_{k=1}^{n}\frac{1}{2}L_{jk}i_ji_k \tag{1.2.48}$$

Going back to the pair of mutually coupled inductors shown in Figure 1.2.9, the flux-linkage relations and the voltage equations for circuits 1 and 2 are given by the following equations, while the resistances associated with the coils are neglected:

$$\lambda_1 = \lambda_{11} + \lambda_{12} = L_{11}i_1 + L_{12}i_2 = L_{11}i_1 + Mi_2 \tag{1.2.49}$$

$$\lambda_2 = \lambda_{21} + \lambda_{22} = L_{21}i_1 + L_{22}i_2 = Mi_1 + L_{22}i_2 \tag{1.2.50}$$

$$v_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + M\frac{di_2}{dt} \tag{1.2.51}$$

$$v_2 = \frac{d\lambda_2}{dt} = M\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} \tag{1.2.52}$$

For the terminal voltage and current assignments shown in Figure 1.2.9, the coil windings are such that the fluxes produced by currents i_1 and i_2 are additive in nature, and in such a case the algebraic sign of the mutual voltage term is positive, as in Equations (1.2.51) and (1.2.52).

In order to avoid drawing detailed sketches of windings showing the sense in which each coil is wound, a *dot convention* is developed, according to which the pair of mutually coupled inductors of Figure 1.2.9 are represented by the system shown in Figure 1.2.10. The notation is such that a current i entering a dotted (undotted) terminal in one coil induces a voltage $M[di/dt]$ with a positive polarity at the dotted (undotted) terminal of the other coil. If the two currents i_1 and i_2 were to be entering (or leaving) the dotted terminals, the adopted convention is such that the fluxes produced by i_1 and i_2 will be aiding each other, and the mutual and self-inductance terms for each terminal pair will have the same sign; otherwise they will have opposite signs.

Although just a pair of mutually coupled inductors are considered here for the sake of simplicity, complicated magnetic coupling situations do occur in practice. For example, Figure 1.2.11 shows the coupling between coils 1 and 2, 1 and 3, and 2 and 3.

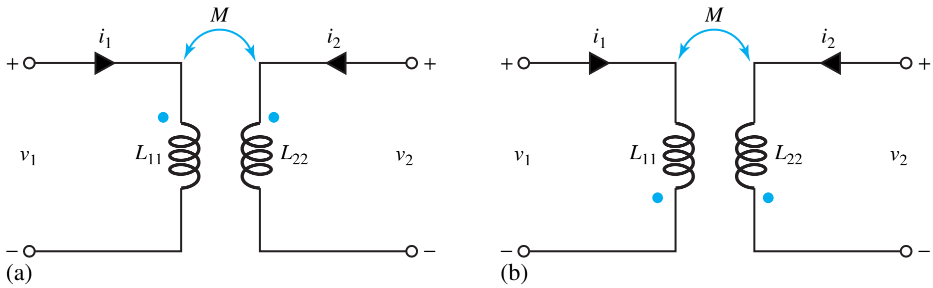


Figure 1.2.10 Dot notation for a pair of mutually coupled inductors. **(a)** Dots on upper terminals. **(b)** Dots on lower terminals.

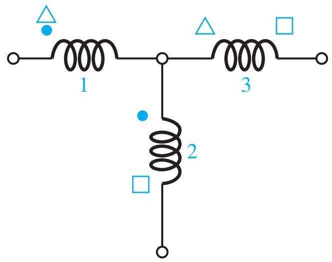


Figure 1.2.11 Polarity markings for complicated magnetic coupling situations.

EXAMPLE 1.2.5

Referring to the circuit of Figure 1.2.8, let

$$L_{11} = L_{22} = 0.1 \text{ H}$$

and

$$M = 10 \text{ mH}$$

Determine v_1 and v_2 if:

(a) $i_1 = 10$ mA and $i_2 = 0$.

(b) $i_1 = 0$ and $i_2 = 10 \sin 100t$ mA.

(c) $i_1 = 0.1 \cos t$ A and $i_2 = 0.3 \sin(t + 30°)$ A.

Also find the energy stored in each of the above cases at $t = 0$.

Solution

$$L_{11} = L_{22} = 0.1\text{H}; \qquad M = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$$

$$v_1 = L_{11}\frac{di_1}{dt} + M\frac{di_2}{dt}; \qquad v_2 = M\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

$$W_m = \text{energy stored} = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$$

(a) Since both i_1 and i_2 are constant and not a function of time,

$v_1 = 0; \qquad v_2 = 0$

$W_m = \frac{1}{2}(0.1)(10 \times 10^{-3})^2 + 0 + 0 = 5 \times 10^{-6} \text{ J} = 5\ \mu\text{J}$

(b) $v_1 = 0 + 10 \times 10^{-3}(10 \times 100 \cos 100t)10^{-3} = 10 \cos 100t \text{ mV}$

$v_2 = 0 + 0.1(10 \times 100 \cos 100t)10^{-3} = 100 \cos 100t \text{ mV} = 0.1 \cos 100t \text{ V}$

$W_m = 0 + 0 + \frac{1}{2}(0.1)(100 \sin^2 100t)10^{-6}; \qquad W_m = 0 \text{ at } t = 0$

(c) $v_1 = 0.1(-0.1 \sin t) + 10 \times 10^{-3}[0.3 \cos(t + 30°)] = -10 \sin t + 3 \cos(t + 30°) \text{ mV}$

$v_2 = 10 \times 10^{-3}(-0.01 \sin t) + 0.1[0.3 \cos(t + 30°)] = -\sin t + 30 \cos(t + 30°) \text{ mV}$

$W_m = \frac{1}{2}(0.1)(0.01 \cos^2 t) + (10 \times 10^{-3})(0.1 \cos t)[0.3 \sin(t + 30°)] + \frac{1}{2}(0.1)$ $[0.09 \sin^2(t + 30°)]; \qquad \text{at } t = 0$

$W_m = \frac{1}{2}(0.1)(0.01) + 10 \times 10^{-3}(0.1)(0.15) + \frac{1}{2}(0.1)\left(\frac{0.09}{4}\right) = 1.775\ \mu\text{J}$

Transformer

A transformer is basically a static device in which two or more stationary electric circuits are coupled magnetically, the windings being linked by a common time-varying magnetic flux. All that is really necessary for transformer action to take place is for the two coils to be so positioned that some of the flux produced by a current in one coil links some of the turns of the other coil. Some air-core transformers employed in communications equipment are no more elaborate than this. However, the construction of transformers utilized in power-system networks is much more elaborate to minimize energy loss, to produce a large flux in the ferromagnetic core by a current in any one coil, and to see that as much of that flux as possible links as many of the turns as possible of the other coils on the core.

An elementary model of a two-winding core-type transformer is shown in Figure 1.2.12. Essentially it consists of two windings interlinked by a mutual magnetic field. The winding that is excited or energized by connecting it to an input source is usually referred to as the *primary* winding, whereas the other, to which the electric load is connected and from which the output energy is taken, is known as the *secondary* winding. Depending on the voltage level at which the winding is operated, the windings are classified as HV (*high voltage*) and LV (*low voltage*) windings. The terminology of *step-up* or *step-down transformer* is also common if the main purpose of the transformer is to raise or lower the voltage level. In a step-up transformer, the primary is a low-voltage winding whereas the secondary is a high-voltage winding. The opposite is true for a step-down transformer.

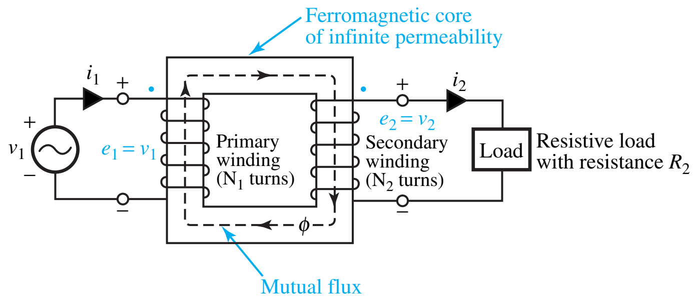


Figure 1.2.12 Elementary model of a two-winding core-type transformer (ideal transformer).

An *ideal transformer* is one that has no losses (associated with iron or copper) and no leakage fluxes (i.e., all the flux in the core links both the primary and the secondary windings). The winding resistances are negligible. While these properties are never actually achieved in practical transformers, they are, however, approached closely. When a time-varying voltage v_1 is applied to the N_1-turn primary winding (assumed to have zero resistance), a core flux ϕ is established and a counter emf e_1 with the polarity shown in Figure 1.2.12 is developed such that e_1 is equal to v_1. Because there is no leakage flux with an ideal transformer, the flux ϕ also links all N_2 turns of the secondary winding and produces an induced emf e_2, according to Faraday's law of induction. Since $v_1 = e_1 = d\lambda_1/dt = N_1\, d\phi/dt$ and $v_2 = e_2 = d\lambda_2/dt = N_2\, d\phi/dt$, it follows from Figure 1.2.12 that

$$\frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a \tag{1.2.53}$$

where a is the *turns ratio*. Thus, in an ideal transformer, voltages are transformed in the direct ratio of the turns. For the case of an ideal transformer, since the instantaneous power input equals the instantaneous power output, it follows that

$$v_1 i_1 = v_2 i_2 \qquad \text{or} \qquad \frac{i_1}{i_2} = \frac{v_2}{v_1} = \frac{N_2}{N_1} = \frac{1}{a} \tag{1.2.54}$$

which implies that currents are transformed in the inverse ratio of the turns.

Equivalent circuits viewed from the source terminals, when the transformer is ideal, are shown in Figure 1.2.13. As seen from Figure 1.2.13(a), since $v_1 = (N_1/N_2)v_2$, $i_1 = (N_2/N_1)i_2$, and $v_2 = i_2 R_L$, it follows that

$$\frac{v_1}{i_1} = \left(\frac{N_1}{N_2}\right)^2 R_L = a^2 R_L = R'_L \tag{1.2.55}$$

where R'_L is the secondary-load resistance *referred to* the primary side. The consequence of Equation (1.2.55) is that a resistance R_L in the secondary circuit can be replaced by an *equivalent* resistance R'_L in the primary circuit in so far as the effect at the source terminals is concerned. The reflected resistance through a transformer can be very useful in *resistance matching* for *maximum power transfer*, as we shall see in the following example. Note that the circuits shown in Figure 1.2.13 are indistinguishable viewed from the source terminals.

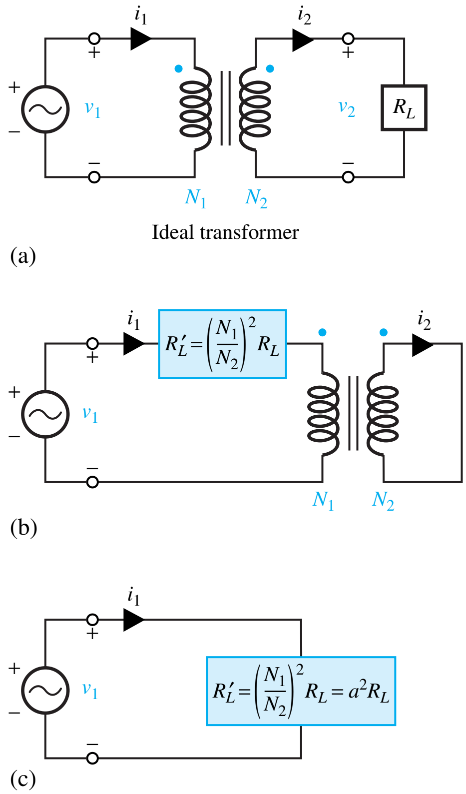


Figure 1.2.13 Equivalent circuits viewed from source terminals when the transformer is ideal.

EXAMPLE 1.2.6

Consider a source of voltage $v(t) = 10\sqrt{2}\ \sin\ 2t$ V, with an internal resistance of 1800 Ω. A transformer that can be considered ideal is used to couple a 50-Ω resistive load to the source.

(a) Determine the primary to secondary turns ratio of the transformer required to ensure maximum power transfer by matching the load and source resistances.

(b) Find the average power delivered to the load.

Solution

By considering a constant voltage source (with a given internal resistance R_S) connected to a variable-load resistance R_L, as shown in Figure E1.2.6(a), for a value of R_L equal to R_S given by Equation (1.2.13), the maximum power transfer to the load resistance would occur when the load resistance is matched with the source resistance.

(a) For maximum power transfer to the load, R_L' (i.e., R_L referred to the primary side of the transformer) should be equal to R_S, which is given to be 1800 Ω. Hence,

$$R_L' = a^2 R_L = 50a^2 = 1800 \qquad \text{or} \qquad a^2 = 36,\ \text{or } a = 6.$$

Thus $N_1/N_2 = 6$ [see Figure E1.2.6(b)].

(b) By voltage division [see Figure E1.2.6(c)] one gets

$$v'_L = \frac{1800}{1800 + 1800}(10\sqrt{2}\ \sin\ 2t) = (5\sqrt{2}\ \sin\ 2t)\ \text{V}$$

which has an rms value of 5 V. Hence, the average power delivered to the load resistance (R'_L or R_L) is

$$P_{\text{av}} = \frac{(V'_{L\ \text{RMS}})^2}{1800} = \frac{25}{1800}\ \text{W} \cong 13.9\ \text{mW}$$

Note that v_L across R_L is $(5\sqrt{2}/6)\ \sin\ 2t$ V, and i_L through R_L is $(5\sqrt{2}/6 \times 50)\ \sin\ 2t$ A. The rms value of i_L is then 5/300 A, and the rms value of v_L is $^5\!/_6$ V. Thus,

$$P_{\text{av}} = \frac{5}{300} \times \frac{5}{6}\ \text{W} \cong 13.9\ \text{mW}$$

which is also the same as

$$P_{\text{av}} = I^2_{L\ \text{RMS}}\ R_L = V^2_{L\ \text{RMS}}/R_L$$

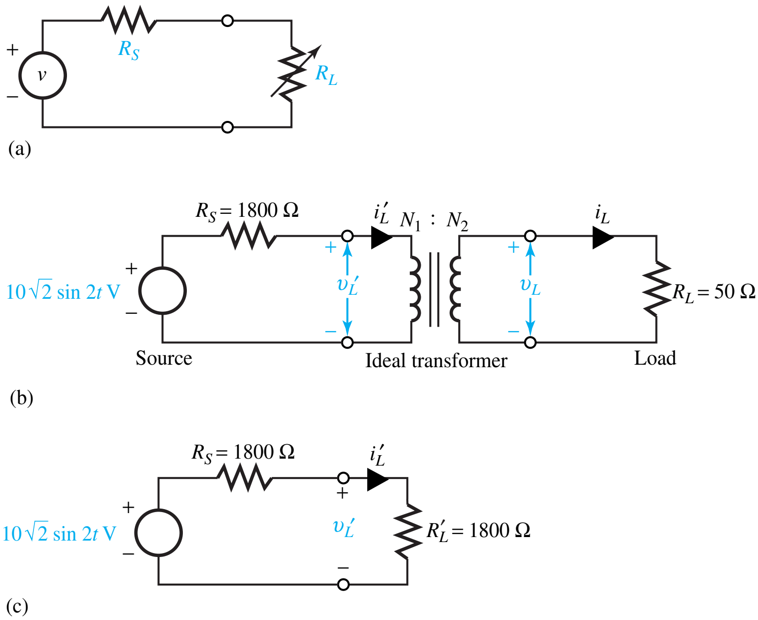


Figure E1.2.6

1.3 KIRCHHOFF'S LAWS

The basic laws that must be satisfied among circuit currents and circuit voltages are known as *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL). These are fundamental for the systematic analysis of electric circuits.

KCL states that, at any node of any circuit and at any instant of time, the sum of all currents entering the node is equal to the sum of all currents leaving the node. That is, the *algebraic* sum of all currents (entering or leaving) at any node is zero, or no node can accumulate or store charge. Figure 1.3.1 illustrates Kirchhoff's current law, in which at node a,

$$i_1 - i_2 + i_3 + i_4 - i_5 = 0 \qquad \text{or} \qquad -i_1 + i_2 - i_3 - i_4 + i_5 = 0$$
$$\text{or} \qquad i_1 + i_3 + i_4 = i_2 + i_5 \tag{1.3.1}$$

Note that so long as one is consistent, it does not matter whether the currents directed toward the node are considered positive or negative.

KVL states that the *algebraic* sum of the voltages (drops or rises) encountered in traversing any *loop* (which is a *closed path* through a circuit in which no electric element or node is encountered more than once) of a circuit in a specified direction must be zero. In other words, the sum of the voltage rises is equal to the sum of the voltage drops in a loop. A loop that contains no other loops is known as a *mesh*. KVL implies that moving charge around a path and returning to the starting point should require no net expenditure of energy. Figure 1.3.2 illustrates the Kirchhoff's voltage law.

For the mesh shown in Figure 1.3.2, which depicts a portion of a network, starting at node a and returning back to it while traversing the closed path *abcdea* in either clockwise or anticlockwise direction, Kirchhoff's voltage law yields

$$-v_1 + v_2 - v_3 - v_4 + v_5 = 0 \qquad \text{or} \qquad v_1 - v_2 + v_3 + v_4 - v_5 = 0$$
$$\text{or} \qquad v_1 + v_3 + v_4 = v_2 + v_5 \tag{1.3.2}$$

Note that so long as one is consistent, it does not matter whether the voltage drops are considered positive or negative. Also notice that the currents labeled in Figure 1.3.2 satisfy KCL at each of the nodes.

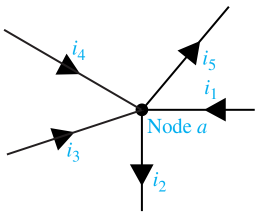


Figure 1.3.1 Illustration of Kirchoff's current law.

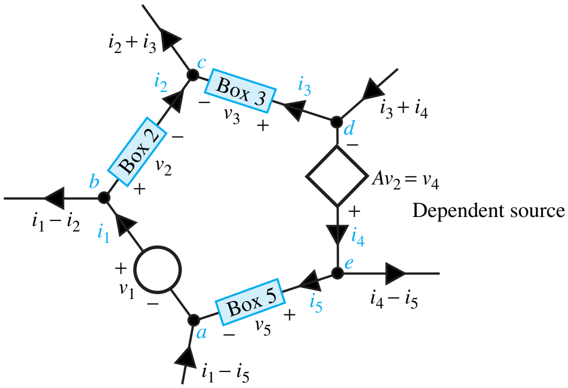


Figure 1.3.2 Illustration of Kirchhoff's voltage law.

In a network consisting of one or more energy sources and one or more circuit elements, the *cause-and-effect* relationship in *circuit theory* can be studied utilizing Kirchhoff's laws and volt-ampere relationships of the circuit elements. While the cause is usually the voltage or current source exciting the circuit, the effect is the voltages and currents existing in various parts of the network.

EXAMPLE 1.3.1

Consider the circuit shown in Figure E1.3.1 and determine the unknown currents using KCL.

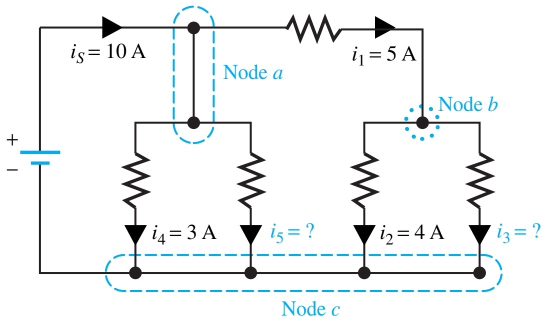


Figure E1.3.1

Solution

Let us assign a + sign for currents entering the node and a − sign for currents leaving the node.

Applying KCL at node a, we get

$$+i_S - i_1 - i_4 - i_5 = 0$$

or

$$10 - 5 - 3 - i_5 = 0 \qquad \text{or} \qquad i_5 = 2 \text{ A}$$

Applying KCL at node b, we get

$$+i_1 - i_2 - i_3 = 0$$

or

$$5 - 4 - i_3 = 0 \qquad \text{or} \qquad i_3 = 1 \text{ A}$$

The student is encouraged to rework this problem by:

(a) Assigning a − sign for currents entering the node and a + sign for currents leaving the node; and

(b) Applying the statement that the sum of the currents entering a node is equal to the sum of the currents leaving that node.

EXAMPLE 1.3.2

For the circuit shown in Figure E1.3.2, use KCL and KVL to determine i_1, i_2, v_{bd} and v_x. Also, find v_{eb}.

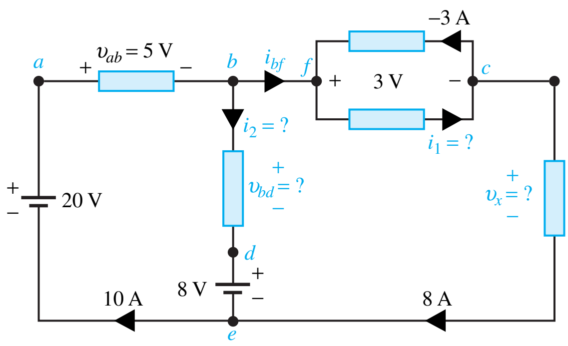


Figure E1.3.2

Solution

Using KCL at node c, we get

$$i_1 = 8 + (-3 = 5 \text{ A}$$

Using KCL at node f, we have

$$i_{bf} = i_1 - (-3) = 5 + 3 = 8 \text{ A}$$

Applying KCL at node b, we get

$$10 = i_2 + i_{bf} = i_2 + 8 \qquad \text{or} \qquad i_2 = 2 \text{ A}$$

Using KVL around the loop *abdea* in the clockwise direction, we have

$$v_{ab} + v_{bd} + v_{de} + v_{ea} = 0$$

or

$$5 + v_{bd} + 8 - 20 = 0 \qquad \text{or} \qquad v_{bd} = 20 - 8 - 5 = 7 \text{ V}$$

Note that in writing KVL equations with + and − polarity symbols, we write the voltage with a positive sign if the + is encountered before the − and with a negative sign if the − is encountered first as we move around the loop.

Applying KVL around the loop *abfcea* in the clockwise direction, we get

$$v_{ab} + v_{bf} + v_{fc} + v_{ce} + v_{ea} = 0$$

or

$$5 + 0 + 3 + v_x + (-20) = 0 \qquad \text{or} \qquad v_x = 20 - 3 - 5 = 12 \text{ V}$$

Note that a direct connection between b and f implies ideal connection, and hence no voltage between these points.

The student is encouraged to rewrite the loop equations by traversing the closed path in the anticlockwise direction.

Noting that $v_{eb} = v_{ed} + v_{db}$, we have

$$v_{eb} = -8 - 7 = -15 \text{ V}$$

Alternatively,

$$v_{eb} = v_{ea} + v_{ab} = -20 + 5 = -15 \text{ V}$$

or

$$v_{eb} = v_{ec} + v_{cf} + v_{fb} = -v_x + v_{cf} + 0 = -12 - 3 = -15 \text{ V}$$

The student should observe that $v_{be} = -v_{eb} = 15$ V and node b is at a higher potential than node e.

EXAMPLE 1.3.3

Referring to Figure 1.3.2, let boxes 2, 3, and 5 consist of a 0.2-H inductor, a 5-Ω resistor, and a 0.1-F capacitor, respectively. Given $A = 5$, and $v_1 = 10 \sin 10t$, $i_2 = 5 \sin 10t$, and $i_3 = 2 \sin 10t - 4 \cos 10t$, find i_5.

Solution

From Equation (1.2.19),

$$v_2 = L\frac{di_2}{dt} = 0.2\frac{d}{dt}(5 \sin 10t) = 10 \cos 10t \text{ V}$$

From Equation (1.2.1),

$$v_3 = Ri_3 = 5(2 \sin 10t - 4 \cos 10t) = 10 \sin 10t - 20 \cos 10t \text{ V}$$

From Equation (1.3.2),

$$\begin{aligned} v_5 &= v_1 + v_3 + v_4 - v_2 = v_1 + v_3 + 5v_2 - v_2 \\ &= 10 \sin 10t + (10 \sin 10t - 20 \cos 10t) + 4(10 \cos 10t) \\ &= 20 \sin 10t + 20 \cos 10t \text{ V} \end{aligned}$$

From Equation (1.2.9),

$$\begin{aligned} i_5 &= C\frac{dv_5}{dt} = 0.1\frac{d}{dt}(20 \sin 10t + 20 \cos 10t) \\ &= 20 \cos 10t - 20 \sin 10t \text{ A} \end{aligned}$$

Note the consistency of voltage polarities and current directions in Figure 1.3.2.

EXAMPLE 1.3.4

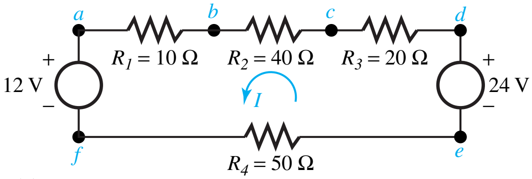
Consider the network shown in Figure E1.3.4(a).

(a) Find the voltage drops across the resistors and mark them with their polarities on the circuit diagram.

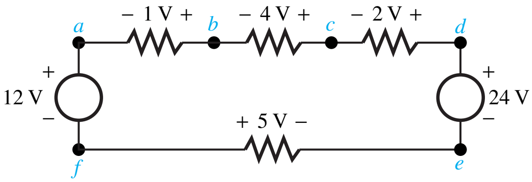
(b) Check whether the KVL is satisfied, and determine V_{bf} and V_{ec}.

(c) Show that the conservation of power is satisfied by the circuit.

Figure E1.3.4



(a)



(b)

Solution

(a) From Ohm's law, the current I is given by

$$I = \frac{24 - 12}{10 + 40 + 20 + 50} = \frac{12}{120} = 0.1 \text{ A}$$

Therefore, voltage drops across the resistors are calculated as follows:

$$\begin{aligned}
V_{ba} &= IR_1 = 0.1 \times 10 = 1 \text{ V} \\
V_{cb} &= IR_2 = 0.1 \times 40 = 4 \text{ V} \\
V_{dc} &= IR_3 = 0.1 \times 20 = 2 \text{ V} \\
V_{fe} &= IR_4 = 0.1 \times 50 = 5 \text{ V}
\end{aligned}$$

These are shown in Figure E1.3.4(b) with their polarities. Note that capital letters are used here for dc voltages and currents.

(b) Applying the KVL for the closed path *edcbafe*, we get

$$-24 + 2 + 4 + 1 + 12 + 5 = 0$$

which confirms that the KVL is satisfied,

$$\begin{aligned}
V_{bf} &= V_{ba} + V_{af} = 1 + 12 = 13 \text{ V} \\
V_{ec} &= V_{ed} + V_{dc} = -24 + 2 = -22 \text{ V}
\end{aligned}$$

(c) Power delivered by 24-V source $= 24 \times 0.1 = 2.4$ W

Power delivered by 12-V source $= -12 \times 0.1 = -1.2$ W

Power absorbed by resistor $R_1 = (0.1)^2 \times 10 = 0.1$ W

Power absorbed by resistor $R_2 = (0.1)^2 \times 40 = 0.4$ W

Power absorbed by resistor $R_3 = (0.1)^2 \times 20 = 0.2$ W

Power absorbed by resistor $R_4 = (0.1)^2 \times 50 = 0.5$ W

Power delivered by sources $= 2.4 - 1.2 = 1.2$ W

Power absorbed by resistors R_1, R_2, R_3, and $R_4 = 0.1 + 0.4 + 0.2 + 0.5 = 1.2$ W

The conservation of power is satisfied by the circuit.

EXAMPLE 1.3.5

Given the network in Figure E1.3.5,

(a) Find the currents through resistors R_1, R_2, and R_3.
(b) Compute the voltage V_1.
(c) Show that the conservation of power is satisfied by the circuit.

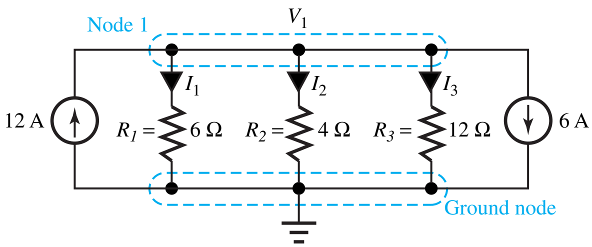


Figure E1.3.5

Solution

(a) Applying the KCL at node 1, we have

$$12 = \frac{V_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1}{R_3} + 6 = V_1\left(\frac{1}{6} + \frac{1}{4} + \frac{1}{12}\right) + 6 = \frac{V_1}{2} + 6$$

Therefore, $V_1 = 12$ V. Then,

$$I_1 = \frac{12}{6} = 2 \text{ A}$$

$$I_2 = \frac{12}{4} = 3 \text{ A}$$

$$I_3 = \frac{12}{12} = 1 \text{ A}$$

(b) $V_1 = 12$ V

(c) Power delivered by 12-A source $= 12 \times 12 = 144$ W

Power delivered by 6-A source $= -6 \times 12 = -72$ W

Power absorbed by resistor $R_1 = I_1^2 R_1 = (2)^2 6 = 24$ W

Power absorbed by resistor $R_2 = I_2^2 R_2 = (3)^2 4 = 36$ W

Power absorbed by resistor $R_3 = I_3^2 R_3 = (1)^2 12 = 12$ W

Power delivered by sources $= 144 - 72 = 72$ W

Power absorbed by resistors R_1, R_2, and $R_3 = 24 + 36 + 12 = 72$ W

The conservation of power is satisfied by the circuit.

EXAMPLE 1.3.6

Consider the network shown in Figure E1.3.6 containing a voltage-controlled source producing the controlled current $i_c = gv$, where g is a constant with units of conductance, and the control voltage happens to be the terminal voltage in this case.

(a) Obtain an expression for $R_{eq} = v/i$.

(b) For (i) $gR = 1/2$, (ii) $gR = 1$, and (iii) $gR = 2$, find R_{eq} and interpret what it means in each case.

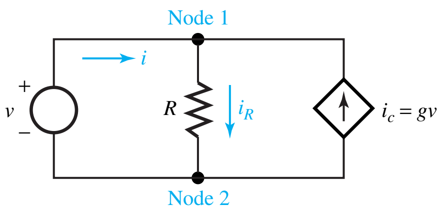


Figure E1.3.6

Solution

(a) Applying the KCL at node 1, we get

$$i + i_c = i_R = \frac{v}{R}$$

Therefore,

$$i = \frac{v}{R} - i_c = \frac{v}{R} - gv = \frac{1 - gR}{R} v$$

Then,

$$R_{eq} = \frac{v}{i} = \frac{R}{1 - gR}$$

(b) (i) For $gR = 1/2$, $R_{eq} = 2R$. The equivalent resistance is greater than R; the internal controlled source provides part of the current through R, thereby reducing the input current i for a given value of v. When $i < v/R$, the equivalent resistance is greater than R.

(ii) For $gR = 1$, $R_{eq} \to \infty$. The internal controlled source provides all of the current through R, thereby reducing the input current i to be zero for a given value of v.

(iii) For $gR = 2$, $R_{eq} = -R$, which is a negative equivalent resistance. This means that the controlled source provides more current than that going through R; the current direction of i is reversed when $v > 0$. However, the relation $v = R_{eq}i$ is satisfied at the input terminals.

1.4 METERS AND MEASUREMENTS

The subject of electrical measurements is such a large one that entire books have been written on the topic. Only a few basic principles will be introduced here. Practical measurements are made with real instruments, which in general disturb the operation of a circuit to some extent when they are connected. Measurements may be affected by *noise*, which is undesirable randomly varying signals.

Voltmeter

In order to measure the potential difference between two terminals or nodes of a circuit, a voltmeter is connected *across* these two points. A practical voltmeter can usually be modeled as a parallel combination of an ideal voltmeter (through which no current flows) and a shunt resistance R_V, as shown in Figure 1.4.1. The internal resistance R_V of an ideal voltmeter is infinite, while its value in practice is of the order of several million ohms. There are what are known as dc voltmeters and ac voltmeters. An ac voltmeter usually measures the rms value of the time-varying voltage.

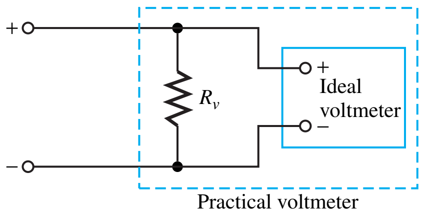


Figure 1.4.1

EXAMPLE 1.4.1

An electromechanical voltmeter with internal resistance of 1 kΩ and an electronic voltmeter with internal resistance of 10 MΩ are used separately to measure the potential difference between A and the ground of the circuit shown in Figure E1.4.1. Calculate the voltages that will be indicated by each of the two instruments and the percentage error in each case.

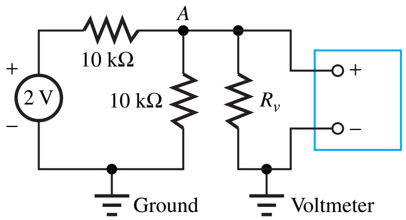


Figure E1.4.1

Solution

When no instrument is connected, by voltage division, $V + AG = 1$ V. With internal resistance R_V of the instrument, the voltmeter reading will be

$$V_A = 2.0\left(\frac{R_V \| 10^4}{R_V \| 10^4 + 10^4}\right) = 2\left(\frac{R_V \cdot 10^4}{R_V + 10^4}\right) \Big/ \left(\frac{R_V \cdot 10^4}{R_V + 10^4} + 10^4\right)$$

$$= 2\frac{R_V \cdot 10^4}{2 \times 10^4\, R_V + 10^8}$$

With $R_V = 1000$,

$$V_A = \frac{2 \times 10^7}{2 \times 10^7 + 10^8} = \frac{2}{2 + 10} = \frac{2}{12} = 0.1667 \text{ V}$$

for which the percent error is

$$\frac{1 - 0.1667}{1} \times 100 = 83.33\%$$

With $R_V = 10^7\ \Omega$,

$$V_A = \frac{2 \times 10^{11}}{2 \times 10^{11} + 10^8} = \frac{2000}{2000 + 1} = 0.9995 \text{ V}$$

for which the percent error is

$$\frac{1 - 0.9995}{1} \times 100 = 0.05\%$$

One can see why electronic voltmeters which have relatively very large internal resistance are often used.

Ammeter

In order to measure the current through a wire or line of a circuit, an ammeter is connected *in series* with the line. A practical ammeter can usually be modeled as a series combination of an ideal ammeter and an internal resistance R_I. The potential difference between the two terminals of an ideal ammeter is zero, which corresponds to zero internal resistance. There are what are known as dc ammeters and ac ammeters. An ac ammeter usually measures the rms value of the time-varying current. Note that for the ammeter to be inserted for measuring current, the circuit has to be broken, whereas for the voltmeter to be connected for measuring voltage, the circuit need not be disassembled.

Multimeters that measure multiple ranges of voltage and current are available in practice. *Ohmmeters* measure the dc resistance by the use of Ohm's law. A multimeter with scales for volts, ohms, and milliamperes is known as VOM. An ohmmeter should not be used to measure the resistance of an electronic component that might be damaged by the sensing current.

Instrument Transformers

These are generally of two types, *potential transformers* (PTs) and *current transformers* (CTs). They are designed in such a way that the former may be regarded as having an ideal potential ratio, whereas the latter has an ideal current ratio. The accuracy of measurement is quite important for ITs that are commonly used in ac circuits to supply instruments, protective relays, and control devices.

PTs are employed to step down the voltage to a suitable level, whereas CTs (connected in series with the line) are used to step down the current for metering purposes. Often the primary of a CT is not an integral part of the transformer itself, but is a part of the line whose current is being measured. In addition to providing a desirable low current in the metering circuit, the CT isolates the meter from the line, which may be at a high potential. Note that the secondary terminals of a CT should *never* be open-circuited under load. The student is encouraged to reason and justify this precaution. One of the most useful instruments for measuring currents in the ampere range is the *clip-on ammeter* combining the CT with one-turn primary and the measurement functions.

Oscilloscope

To measure time-varying signals (voltages and currents), an instrument known as an *oscilloscope* is employed. It can be used as a practical electronic voltmeter which displays a graph of voltage as a function of time. Such a display allows one not only to read off the voltage at any instant of time, but also to observe the general behavior of the voltage as a function of time. The horizontal and vertical scales of the display are set by the oscilloscope's controls, such as 5 ms per each horizontal division and 50 V per each vertical division. For periodic waveforms, the moving light spot repeatedly graphs the same repetitive shape, and the stationary waveform is seen. For nonperiodic cases, a common way of handling is to cause the oscilloscope to make only one single graph, representing the voltage over a single short time period. This is known as *single-sweep operation*. Since the display lasts for only a very short time, it may be photographed for later inspection.

Digital meters are generally more accurate and can be equipped with more scales and broader ranges than *analog meters*. On the other hand, analog meters are generally less expensive and give an entire range or scale of reading, which often could be very informative. A digital oscilloscope represents the combination of analog and digital technologies. By digital sampling techniques, the oscilloscope trace is digitized and stored in the digital memory included with the digital oscilloscope. Digital oscilloscopes are generally more costly than analog ones, but their capability in the analysis and processing of signals is vastly superior.

Wheatstone Bridge

Null measurements are made with bridge circuits and related configurations. They differ from direct measurements in that the quantity being measured is compared with a known reference quantity. The balancing strategy avoids undesirable interaction effects and generally results in more accurate measurement than the direct one.

By far the most common is the Wheatstone bridge designed for precise measurement of resistance. Figure 1.4.2 shows the basic circuit in which the measurement of an unknown resistance R_x is performed by balancing the variable resistances R_a and R_b until no current flows through meter A. Under this null condition,

$$R_x = \frac{R_a}{R_b} \cdot R_s \tag{1.4.1}$$

where R_s is the known standard resistance. There are other bridge-circuit configurations to measure inductance and capacitance. Typical instruments utilizing bridge circuits are found in strain gauges measuring stress and in temperature measuring systems with thermocouples and thermistors.

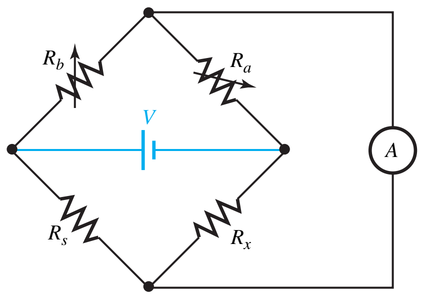


Figure 1.4.2 Basic Wheatstone bridge circuit for resistance measurement.

EXAMPLE 1.4.2

Redraw the Wheatstone bridge circuit of Figure 1.4.2 and show that Equation (1.4.1) holds good for the null condition when the meter A reads zero current.

Solution

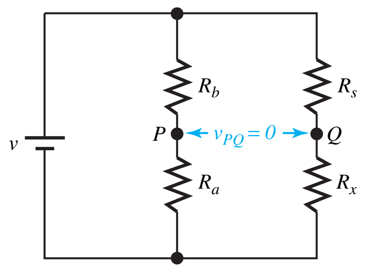


Figure E1.4.2 Figure 1.4.2 redrawn.

Under null condition, $V_{PQ} = 0$, or P and Q are at the same potential. Using the voltage division principle,

$$\left(\frac{V}{R_b + R_a}\right) R_b = \left(\frac{V}{R_s + R_x}\right) R_s$$

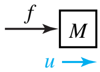
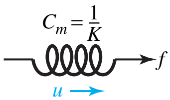
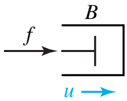
yielding $R_x R_b = R_a R_s$, which is the same as Equation (1.4.1).

1.5 ANALOGY BETWEEN ELECTRICAL AND OTHER NONELECTRIC PHYSICAL SYSTEMS

Systems such as those encountered in mechanics, thermodynamics, and hydraulics can be represented by analogous electric networks, from the response of which the system characteristics can be investigated. The *analogy*, of course, is a mathematical one: that is, two systems are analogous to each other if they are described by similar equations. The analogous electric quantities for a mechanical system are listed in Table 1.5.1.

Consider a tank filled with water, as shown in Figure 1.5.1, with input flow rate F_i and output flow rate $F_o = h/R$, where h is the fluid level or head and R is related to the diameter of the pipe, denoting the fluid resistance. Let A be the cross-sectional area of the tank. We may think of the fluid as being analogous to charge, and the fluid flow as being analogous to current. Then, in effect, the water tank acts as a capacitor storing charge, which is fluid in this case. This analogy is illustrated in Table 1.5.2.

TABLE 1.5.1 Mechanical-Electrical Analogs

Quantity	Unit	Symbol	Mathematical Relation	Force–Current Analog	Force–Voltage Analog
Force	N	$f(t)$	$\cdots$	$i(t)$	$v(t)$
Velocity	m/s	$u(t)$	$\cdots$	$v(t)$	$i(t)$
Mass	kg		$f = M\frac{du}{dt}$ (Newton's law)	C	L
Compliance (= 1/stiffness)	m/N		$f = \frac{1}{C_m}\int u\,dt$ (Hooke's law)	L	C
Viscous friction or damping	$N \cdot s/m$		$f = Bu$	$G = 1/R$	R

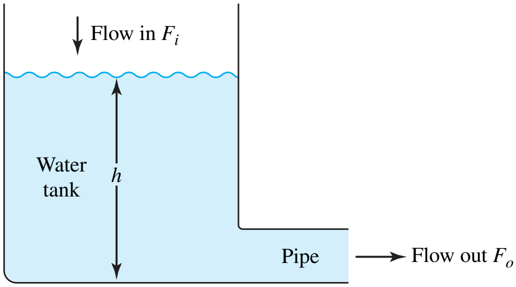


Figure 1.5.1 Simple hydraulic system.

TABLE 1.5.2 Analogy Between Electrical and Hydraulic Systems

Quantity	Hydraulic System	Electrical System
Flow	Output flow rate F_o	Current i
Potential	Fluid level h	Voltage v
Resistance	Fluid resistance R	Electrical resistance R
Energy storage element	Fluid storage parameter A	Capacitance C
Volume of fluid (or charge)	$V = Ah$	$q = Cv$

Next, let us consider heat flow from an enclosure with a heating system to the outside of the enclosure, depending upon the temperature difference ΔT between the inside and outside of the enclosure. The heat capacity of the enclosure is analogous to capacitance, in the sense that the enclosure retains part of the heat produced by the heating system. One can then infer the analogy between electrical and thermal systems given in Table 1.5.3.

The heat flow, per Newton's law of cooling in a very simplified form, can be considered to be proportional to the rate of change of temperature with respect to distance. An approximate linear relationship between heat flow and change in temperature can be expressed as

TABLE 1.5.3 Analogy Between Thermal and Electrical Systems

Thermal System	Electrical System
Heat	Charge
Heat flow	Current
Temperature difference	Voltage
Ambient temperature	Ground reference
Heat capacity	Capacitance
Thermal resistance	Electrical resistance

$$\text{Heat flow} \cong \frac{k}{\Delta x}\Delta T \tag{1.5.1}$$

where k is a constant, and $k/\Delta x$ in thermal systems is analogous to conductance in electrical systems. Then Newton's law of cooling, in a very simplified form, can be seen to be a thermal version of Ohm's law.

1.6 LEARNING OBJECTIVES

The *learning objectives* of this chapter are summarized here, so that the student can check whether he or she has accomplished each of the following.

- Review of basic electrical quantities.
- Application of Coulomb's law, Ampere's law, and the Biot–Savart law.
- Energy and power computations in a circuit consisting of a source and a load.
- Calculation of average and RMS values for periodic waveforms, and time constant for exponential waveforms.
- i–v relationships for ideal resistors, capacitors, and inductors; duality principle.
- Reduction of series and parallel combinations of resistors, capacitors, and inductors.
- Solution of simple voltage and current divider circuits.
- Computation of power absorbed by a resistor, and energy stored in a capacitor or inductor.
- Maximum power transfer and matched load.
- Volt-ampere equations and energy stored in coupled inductors.
- Ideal transformer and its properties.
- Application of Kirchhoff's current and voltage laws to circuits.
- Measurement of basic electrical parameters.
- Analogy between electrical and other nonelectric physical systems.

1.7 PRACTICAL APPLICATION: A CASE STUDY

Resistance Strain Gauge

Mechanical and civil engineers routinely employ the dependence of resistance on the physical dimensions of a conductor to measure strain. A strain gauge is a device that is bonded to the surface of an object, and whose resistance varies as a function of the surface strain experienced by the object. Strain gauges can be used to measure strain, stress, force, torque, and pressure.

The resistance of a conductor with a circular cross-sectional area A, length l, and conductivity σ is given by Equation (1.2.2),

$$R = \frac{l}{\sigma A}$$

Depending on the compression or elongation as a consequence of an external force, the length changes, and hence the resistance changes. The relationship between those changes is given by the gauge factor G,

$$G = \frac{\Delta R/R}{\Delta l/l}$$

in which the factor $\Delta l/l$, the fractional change in length of an object, is known as the strain. Alternatively, the change in resistance due to an applied strain $\varepsilon (= \Delta l/l)$ is given by

$$\Delta R = R_0 G \varepsilon$$

where R_0 is the zero-strain resistance, that is, the resistance of the strain gauge under no strain. A typical gauge has $R_0 = 350\ \Omega$ and $G = 2$. Then for a strain of 1%, the change in resistance is $\Delta R = 7\ \Omega$. A Wheatstone bridge as presented in Section 1.4 is usually employed to measure the small resistance changes associated with precise strain determination.

A typical strain gauge, shown in Figure 1.7.1, consists of a metal foil (such as nickel–copper alloy) which is formed by a photoetching process in multiple conductors aligned with the direction of the strain to be measured. The conductors are usually bonded to a thin backing made out of a tough flexible plastic. The backing film, in turn, is attached to the test structure by a suitable adhesive.

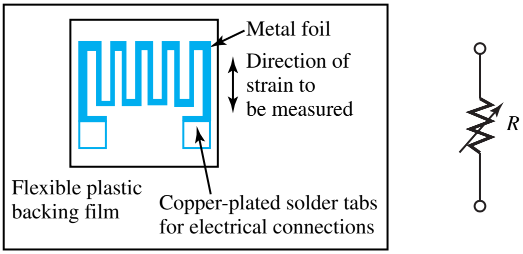


Figure 1.7.1 Resistance strain gauge and circuit symbol.

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PROBLEMS

1.1.1 Consider two 1-C charges separated by 1 m in free space. Show that the force exerted on each is about one million tons.

***1.1.2** Point charges, each of $\sqrt{4\pi\varepsilon_0}$ C, are located at the vertices of an equilateral triangle of side a. Determine the electric force on each charge.

1.1.3 Two charges of equal magnitude 5 μC but opposite sign are separated by a distance of 10 m. Find the net force experienced by a positive charge $q = 2\ \mu$C that is placed midway between the two charges.

1.1.4 The electric field intensity due to a point charge in free space is given to be

$$(-\bar{a}_x - \bar{a}_y + \bar{a}_z)/\sqrt{12}\text{V/m at } (0,0,1)$$
$$\text{and} \qquad 6\,\bar{a}_z \text{ at } (2,2,0)$$

Determine the location and the value of the point charge.

*Complete solutions for problems marked with an asterisk can be found on the CD-ROM packaged with this book.

1.1.5 A wire with $n = 10^{30}$ electrons/m^3 has an area of cross section $A = 1$ mm^2 and carries a current $i = 50$ mA. Compute the number of electrons that pass a given point in 1 s, and find their average velocity.

1.1.6 A beam containing two types of charged particles is moving from A to B. Particles of type I with charge $+3q$, and those of type II with charge $-2q$ (where $-q$ is the charge of an electron given by -1.6×10^{-19} C) flow at rates of 5×10^{15}/s and 10×10^{15}/s, respectively. Evaluate the current flowing in the direction from B to A.

1.1.7 A charge $q(t) = 50 + 1.0t$ C flows into an electric component. Find the current flow.

***1.1.8** A charge variation with time is given in Figure P1.1.8. Draw the corresponding current variation with time.

1.1.9 A current $i(t) = 20 \cos(2\pi \times 60)t$ A flows through a wire. Find the charge flowing, and the number of electrons per second that are passing some point in the wire.

1.1.10 Consider a current element $I_1\, d\bar{l}_1 = 10\, dz\bar{a}_z$ kA located at (0,0,1) and another $I_2\, d\bar{l}_2 = 5dx\bar{a}_x$ kA located at (0,1,0). Compute $d\bar{F}_{21}$ and $d\bar{F}_{12}$ experienced by elements 1 and 2, respectively.

1.1.11 Given $\bar{B} = (y\bar{a}_x - x\bar{a}_y)/(x^2 + y^2)$ T, determine the magnetic force on the current element $I\, d\bar{l} = 5 \times 0.001\bar{a}_z$ A located at (3,4,2).

***1.1.12** In a magnetic field $\bar{B} = B_0(\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z)$ T at a point, let a test charge have a velocity of $v_0(\bar{a}_x + \bar{a}_y - \bar{a}_z)$. Find the electric field $\bar{E}$ at that point if the acceleration experienced by the test charge is zero.

1.1.13 Consider an infinitely long, straight wire (in free space) situated along the z-axis and carrying current of I A in the positive z-direction. Obtain an expression for $\bar{B}$ everywhere. (*Hint:* Consider a circular coordinate system and apply the Biot–Savart law.)

1.1.14 A magnetic force exists between two adjacent, parallel current-carrying wires. Let I_1 and I_2 be the currents carried by the wires, and r the separation between them. Making use of the result of Problem 1.1.13, find the force between the wires.

1.1.15 A point charge $Q_1 = -5$ nC is located at (6, 0, 0). Compute the voltage v_{ba} between two points a(1, 0, 0) and b(5, 0, 0). Comment on whether point a is at a higher potential with respect to point b.

1.1.16 A charge of 0.1 C passes through an electric source of 6 V from its negative to its positive terminals. Find the change in energy received by the charge. Comment on whether the charge has gained or lost energy, and also on the sign to be assigned to the change of energy.

1.1.17 The voltage at terminal a relative to terminal b of an electric component is $v(t) = 20 \cos 120\pi t$ V. A current $i(t) = -4 \sin 120\pi t$ A flows into terminal a. From time t_1 to t_2, determine the total energy flowing into the component. In particular, find the energy absorbed when $t_2 = t_1 + {}^1\!/_{15}$.

***1.1.18** Obtain the instantaneous power flow into the component of Problem 1.1.17, and comment on the sign associated with the power.

1.1.19 A residence is supplied with a voltage $v(t) = 110\sqrt{2} \cos 120\pi t$ V and a current $i(t) = 10\sqrt{2} \cos 120\pi t$ A. If an electric meter is used to measure the average power, find the meter reading, assuming that the averaging is done over some multiple of ${}^1\!/_{60}$ s.

1.1.20 A 12-V, 115-Ah automobile storage battery is used to light a 6-W bulb. Assuming the battery to be a constant-voltage source, find how long the bulb can be lighted before the battery is completely discharged. Also, find the total energy stored in the battery before it is connected to the bulb.

1.2.1 In English units the conductor cross-sectional area is expressed in circular mils (cmil). A circle with diameter d mil has an area of $(\pi/4)d^2$ sq. mil,

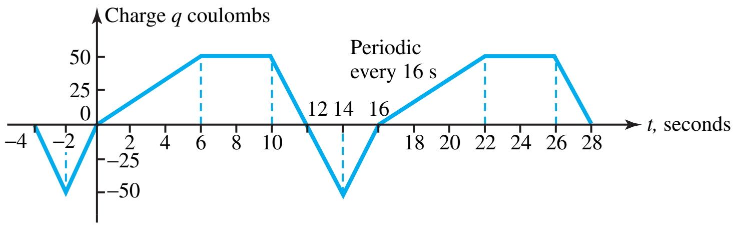

Figure P1.1.8

or d^2 cmil. The handbook for aluminum electrical conductors lists a dc resistance of 0.01558 Ω per 1000 ft at 20°C for Marigold conductor, whose size is 1113 kcmil.

(a) Verify the dc resistance assuming an increase in resistance of 3% for spiraling of the strands.

(b) Calculate the dc resistance at 50°C, given that the temperature constant for aluminum is 228.1°C.

(c) If the 60-Hz resistance of 0.0956 Ω/mile at 50°C is listed in the handbook, determine the percentage increase due to skin effect or frequency.

1.2.2 MCM is the abbreviation for 1 kcmil. (See Problem 1.2.1 for a definition of cmil.) Data for commercial-base aluminum electrical conductors list a 60-Hz resistance of 0.0880 Ω/km at 75°C for a 795-MCM conductor.

(a) Determine the cross-sectional conducting area of this conductor in m^2.

(b) Calculate the 60-Hz resistance of this conductor in Ω/km at 50°C, given a temperature constant of 228.1°C for aluminum.

***1.2.3** A copper conductor has 12 strands with each strand diameter of 0.1328 in. For this conductor, find the total copper cross-sectional area in cmil (see Problem 1.2.1 for definition of cmil), and calculate the dc resistance at 20°C in (ohms/km), assuming a 2% increase in resistance due to spiraling.

1.2.4 A handbook lists the 60-Hz resistance at 50°C of a 900-kcmil aluminum conductor as 0.1185 Ω/mile. If four such conductors are used in parallel to form a line, determine the 60-Hz resistance of this line in Ω/km per phase at 50°C.

1.2.5 Determine R_{eq} for the circuit shown in Figure P1.2.5 as seen from terminals A–B.

1.2.6 Viewed from terminals A–B, calculate R_{eq} for the circuit given in Figure P1.2.6.

1.2.7 Find R_{eq} for the circuit of Figure P1.2.7.

***1.2.8** Determine R_{eq} for the circuit of Figure P1.2.8 as seen from terminals A–B.

1.2.9 A greatly simplified model of an audio system is shown in Figure P1.2.9. In order to transfer maximum power to the speaker, one should select equal values of R_L and R_S. Not knowing that

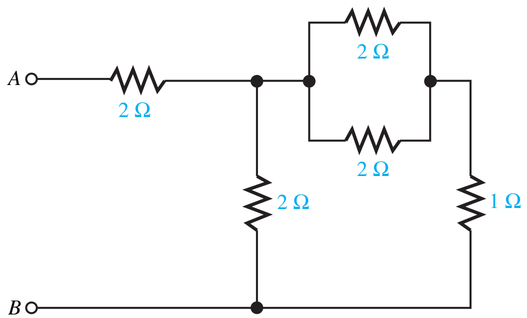

Figure P1.2.5

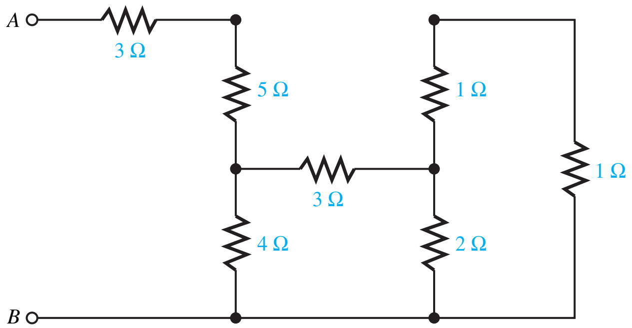

Figure P1.2.6

the internal resistance of the amplifier is $R_S = 8\,\Omega$, one has connected a mismatched speaker with $R_L = 16\,\Omega$. Determine how much more power could be delivered to the speaker if its resistance were matched to that of the amplifier.

1.2.10 For the circuit of Figure P1.2.10:

(a) Find an expression for the power absorbed by the load as a function of R_L.

(b) Plot the power dissipated by the load as a function of the load resistance, and determine the value of R_L corresponding to the maximum power absorbed by the load.

1.2.11 A practical voltage source is represented by an ideal voltage source of 30 V along with a series internal source resistance of 1.2 Ω. Compute the smallest load resistance that can be connected to the practical source such that the load voltage

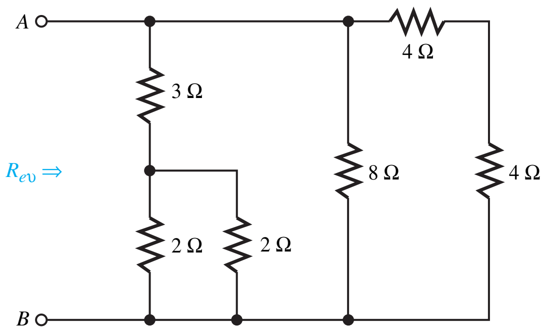

Figure P1.2.7

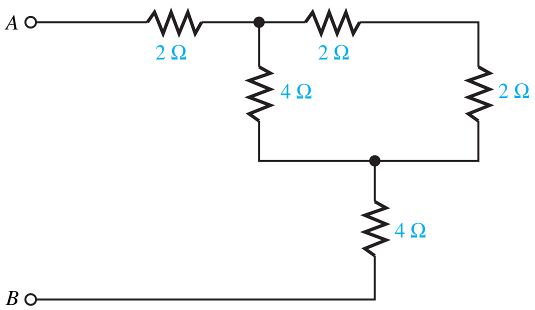

Figure P1.2.8

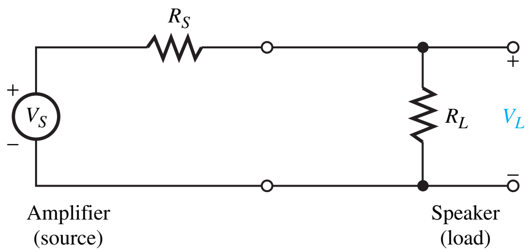

Figure P1.2.9

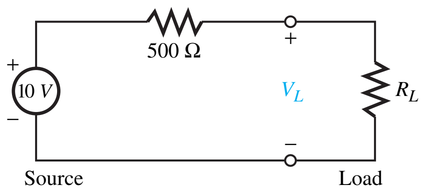

Figure P1.2.10

would not drop by more than 2% with respect to the source open-circuit voltage.

***1.2.12** A practical current source is represented by an ideal current source of 200 mA along with a shunt internal source resistance of 12 kΩ. Determine the percentage drop in load current with respect to the source short-circuit current when a 200-Ω load is connected to the practical source.

1.2.13 Let $v(t) = V_{\max} \cos \omega t$ be applied to (a) a pure resistor, (b) a pure capacitor (with zero initial capacitor voltage, and (c) a pure inductor (with zero initial inductor current). Find the average power absorbed by each element.

1.2.14 If $v(t) = 120\sqrt{2} \sin 2\pi \times 60t$ V is applied to terminals A–B of problems 1.2.5, 1.2.6, 1.2.7, and 1.2.8, determine the power in kW converted to heat in each case.

1.2.15 With a direct current of I A, the power expended as heat in a resistor of $R\,\Omega$ is constant, independent of time, and equal to I^2R. Consider Problem 1.2.14 and find in each case the effective value of the current to give rise to the same heating effect as in the ac case, thereby justifying that the rms value is also known as the effective value for periodic waveforms.

1.2.16 Consider Problem 1.2.14 and obtain in each case a replacement of the voltage source by an equivalent current source at terminals A–B.

1.2.17 Consider Problem 1.2.5. Let $V_{AB} = 120$ V (rms). Show the current and voltage distribution clearly in all branches of the original circuit configuration.

***1.2.18** Determine the voltages V_x using voltage division and equivalent resistor reductions for the circuits shown in Figure P1.2.18.

1.2.19 Find the currents I_x using current division and equivalent resistor reductions for the networks given in Figure P1.2.19.

1.2.20 Considering the circuit shown in Figure P1.2.20, sketch $v(t)$ and the energy stored in the capacitor as a function of time.

1.2.21 For the capacitor shown in Figure P1.2.21 connected to a voltage source, sketch $i(t)$ and $w(t)$.

***1.2.22** The energy stored in a 2-μF capacitor is given by $w_c(t) = 9e^{-2t}$ μJ for $t \geq 0$. Find the capacitor voltage and current at $t = 1$ s.

1.2.23 For a parallel-plate capacitor with plates of area A m^2 and separation d m in air, the capacitance in farads may be computed from the approximate relation

$$C \approx \varepsilon_0 \frac{A}{d} = \frac{8.854 \times 10^{-12} A}{d}$$

Compute the area of each plate needed to develop $C = 1$ pF for $d = 1$ m. (You can appreciate why large values of capacitance are constructed as electrolytic capacitors, and modern integrated-circuit technology is utilized to obtain a wide variety of capacitance values in an extremely small space.)

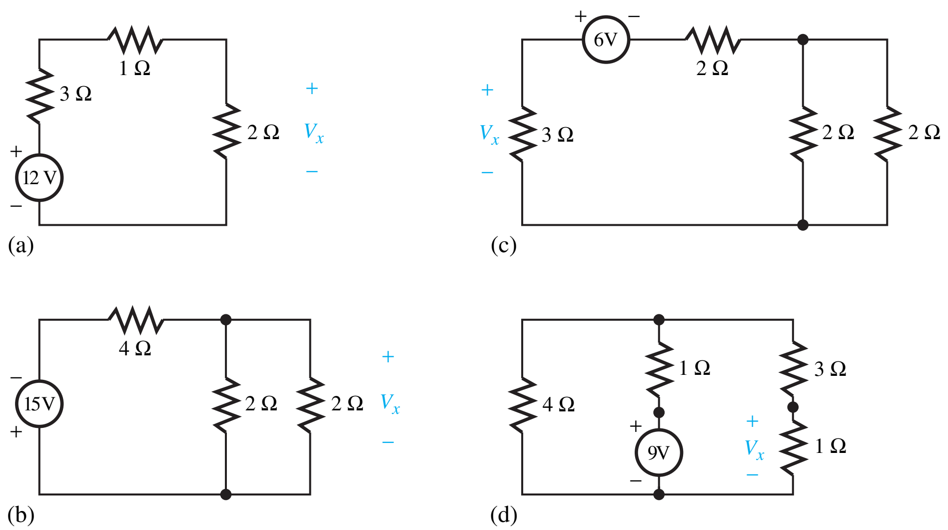

Figure P1.2.18

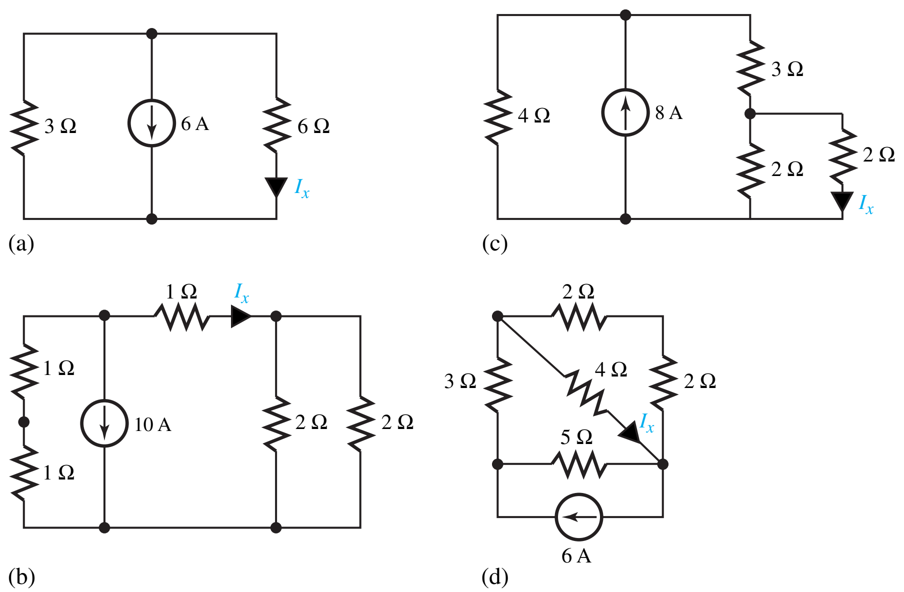

Figure P1.2.19

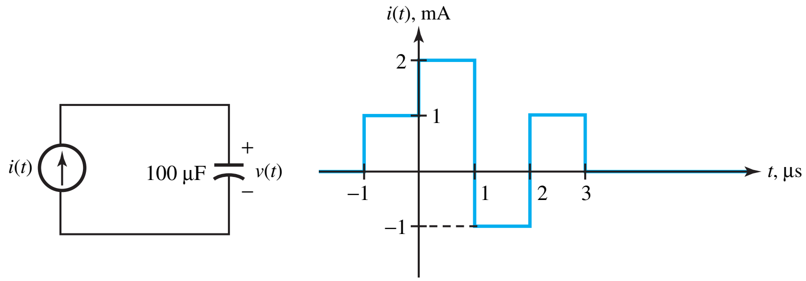

Figure P1.2.20

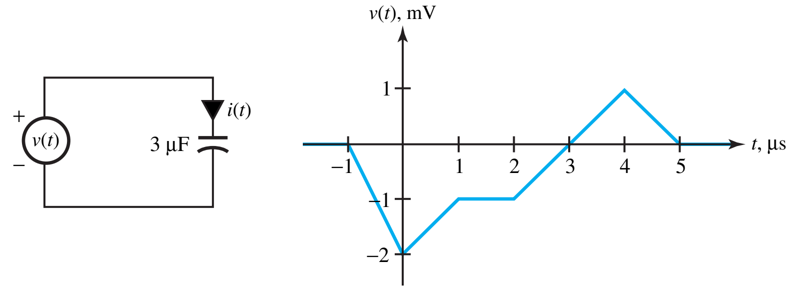

Figure P1.2.21

1.2.24 Determine the equivalent capacitance at terminals A–B for the circuit configurations shown in Figure P1.2.24.

1.2.25 For the circuit shown in Figure P1.2.25, sketch $i(t)$ and $w(t)$. See Problem 1.2.21 and check whether the duality principle is satisfied.

1.2.26 For the circuit given in Figure P1.2.26, sketch $v(t)$ and $w(t)$. See Problem 1.2.20 and check whether the duality principle is satisfied.

***1.2.27** The energy stored in a 2-μH inductor is given by $w_L(t) = 9e^{-2t}$ μJ for $t \geq 0$. Find the inductor current and voltage at $t = 1$ s. Compare the results of this problem with those of Problem 1.2.22 and comment.

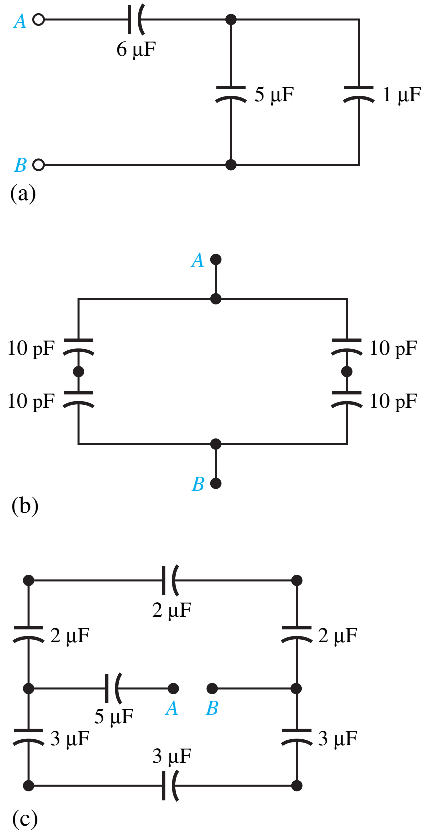

Figure P1.2.24

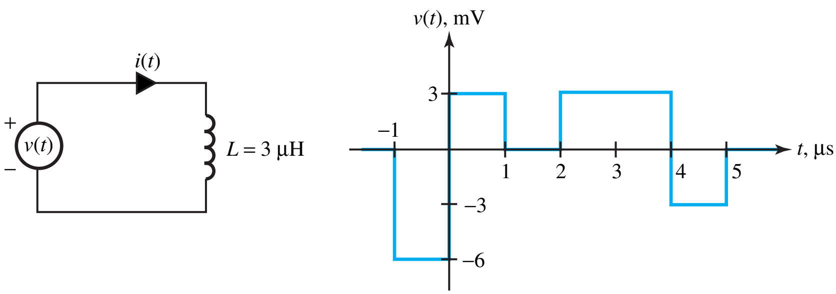

Figure P1.2.25

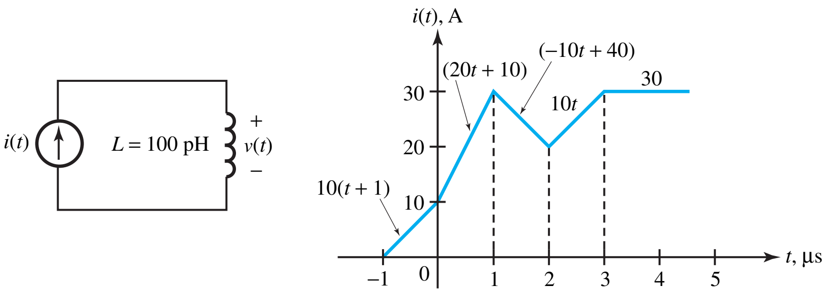

Figure P1.2.26

1.2.28 The inductance per unit length in H/m for parallel-plate infinitely long conductors in air is given by $L = \mu_0 d/w = 4\pi \times 10^{-7} d/w$, where d and w are in meters. Compute L (per unit length) for $d = 1$ m and $w = 0.113$ m. See Problem 1.2.23 and show that the product of inductance (per unit length) and capacitance (per unit length) is $\mu_0 \varepsilon_0$.

1.2.29 Determine the duals for the circuit configurations of Problem 1.2.24 and determine the equivalent inductance at terminals A–B for each case.

1.2.30 Consider a pair of coupled coils as shown in Figure 1.2.10 of the text, with currents, voltages, and polarity dots as indicated. Show that the mutual inductance is $L_{12} = L_{21} = M$ by following these steps:

(a) Starting at time t_0 with $i_1(t_0) = i_2(t_0) = 0$, maintain $i_2 = 0$ and increase i_1 until, at time t_1, $i_1(t_1) = I_1$ and $i_2(t_1) = 0$. Determine the energy accumulated during this time. Now maintaining $i_1 = I_1$, increase i_2 until at time t_2, $i_2(t_2) = I_2$. Find the corresponding energy accumulated and the total energy stored at time t_2.

(b) Repeat the process in the reverse order, allowing the currents to reach their final values. Compare the expressions obtained for the total energy stored and obtain the desired result.

1.2.31 For the coupled coils shown in Figure P1.2.31, a dot has been arbitrarily assigned to a terminal as indicated. Following the dot convention presented in the text, place the other dot in the remaining coil and justify your answer with an explanation. Comment on whether the polarities are consistent with Lenz's law.

***1.2.32** For the configurations of the coupled coils shown in Figure P1.2.32, obtain the voltage equations for v_1 and v_2.

1.2.33 The self-inductances of two coupled coils are L_{11} and L_{22}, and the mutual inductance between them is M. Show that the effective inductance of the two coils in series is given by

$$L_{\text{series}} = L_{11} + L_{22} \pm 2M$$

and the effective inductance of the two coils in parallel is given by

$$L_{\text{parallel}} = \frac{L_{11}L_{22} - M^2}{L_{11} \mp 2M + L_{22}}$$

Specify the conditions corresponding to different signs of the term $2M$.

1.2.34 For the coupled inductors shown in Figure P1.2.34, neglecting the coil resistances, write the volt-ampere relations.

1.2.35 Consider an amplifier as a voltage source with an internal resistance of 72 Ω. Find the turns ratio of the ideal transformer such that maximum power is delivered when the amplifier is connected to an 8-Ω speaker through an $N_1 : N_2$ transformer.

1.2.36 For the circuit shown in Figure P1.2.36, determine $v_{\text{out}}(t)$.

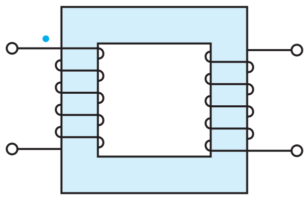

Figure P1.2.31

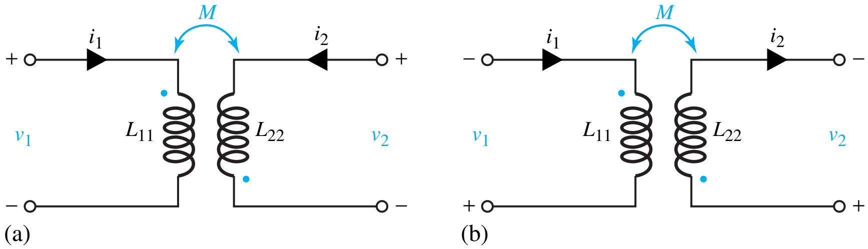

Figure P1.2.32

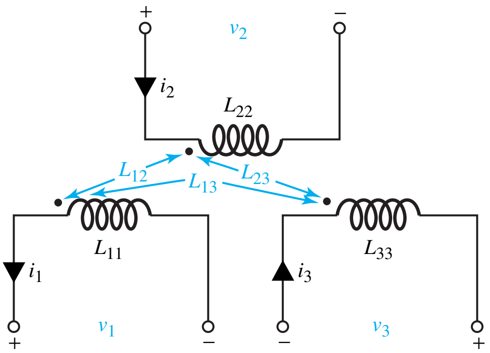

Figure P1.2.34

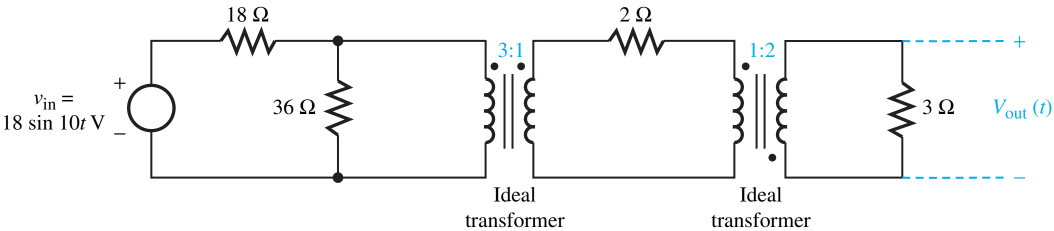

Figure P1.2.36

***1.2.37** A 60-Hz, 100-kVA, 2400/240-V (rms) transformer is used as a step-down transformer from a transmission line to a distribution system. Consider the transformer to be ideal.

(a) Find the turns ratio.

(b) What secondary load resistance will cause the transformer to be fully loaded at rated voltage (i.e., delivering the rated kVA), and what is the corresponding primary current?

(c) Determine the value of the load resistance referred to the primary side of the transformer.

1.2.38 A transformer is rated 10 kVA, 220:110 V (rms). Consider it an ideal transformer.

(a) Compute the turns ratio and the winding current ratings.

(b) If a 2-Ω load resistance is connected across the 110-V winding, what are the currents in the high-voltage and low-voltage windings when rated voltage is applied to the 220-V primary?

(c) Find the equivalent load resistance referred to the 220-V side.

1.3.1 Some element voltages and currents are given in the network configuration of Figure P1.3.1. Determine the remaining voltages and currents. Also calculate the power delivered to each element as well as the algebraic sum of powers of all elements, and comment on your result while identifying sources and sinks.

***1.3.2** Calculate the voltage v in the circuit given in Figure P1.3.2.

1.3.3 Determine v, i, and the power delivered to elements in the network given in Figure P1.3.3. Check whether conservation of power is satisfied by the circuit.

1.3.4 For a part of the network shown in Figure P1.3.4, given that $i_1 = 4$ A; $i_3(t) = 5e^{-t}$, and $i_4(t) = 10 \cos 2t$, find v_1, v_2, v_3, v_4, i_2, and i_5.

1.3.5 For the circuit given in Figure P1.3.5, given that $V_{AC} = 10$ V and $V_{BD} = 20$ V, determine V_1 and V_2. Show that the conservation of power is satisfied by the circuit.

***1.3.6** The current sources in Figure P1.3.6 are given to be $I_A = 30$ A and $I_B = 50$ A. For the values of $R_1 = 20\ \Omega$, $R_2 = 40\ \Omega$, and $R_3 = 80\ \Omega$, find:

(a) The voltage V.

(b) The currents I_1, I_2, and I_3.

(c) The power supplied by the current sources and check whether conservation of power is satisfied.

1.3.7 Show how the conservation of power is satisfied by the circuit of Figure P1.3.7.

1.3.8 Given that $V_0 = 10$ V, determine I_S in the circuit drawn in Figure P1.3.8.

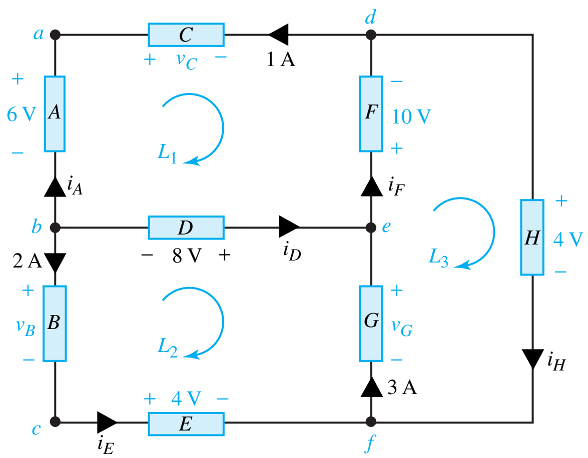

Figure P1.3.1

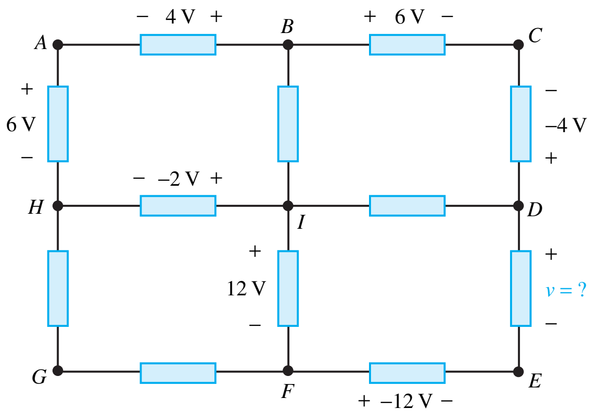

Figure P1.3.2

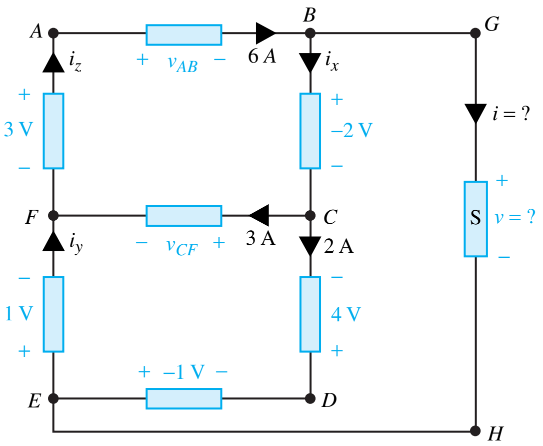

Figure P1.3.3

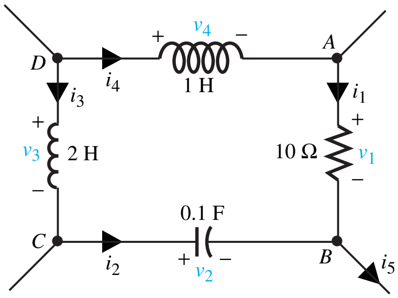

Figure P1.3.4

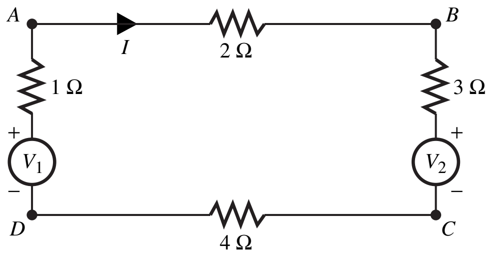

Figure P1.3.5

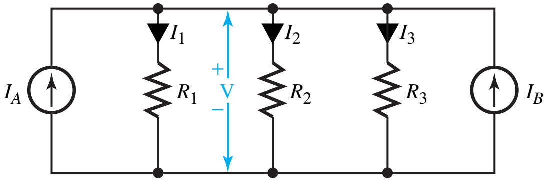

Figure P1.3.6

1.3.9 Consider the circuit shown in Figure P1.3.9.

(a) Given $v(t) = 10e^{-t}$ V, find the current source $i_s(t)$ needed.

(b) Given $i(t) = 10e^{-t}$ A, find the voltage source $v_s(t)$ needed.

1.3.10 An operational amplifier stage is typically represented by the circuit of Figure P1.3.10. For the values given, determine V_{out} and the power supplied by the 2.5-V source.

1.4.1 A voltmeter with a full scale of 100 V has a probable error of 0.1% of full scale. When this meter is employed to measure 100 V, find the percent of probable error that can exist.

1.4.2 A current of 65 A is measured with an analog ammeter having a probable error of ±0.5% of full scale of 100 A. Find the maximum probable percentage error in the measurement.

1.4.3 Error specifications on a 10-A digital ammeter are given as 0.07% of the reading, 0.05% of full scale, 0.005% of the reading per degree Celsius, and 0.002% of full scale per degree Celsius. The 10-A analog meter error specification is given as 0.5% of full scale and 0.001% of the reading per

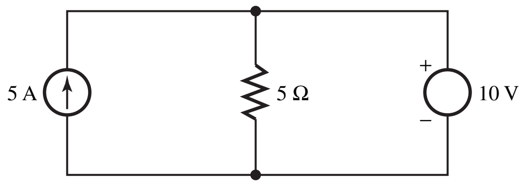

Figure P1.3.7

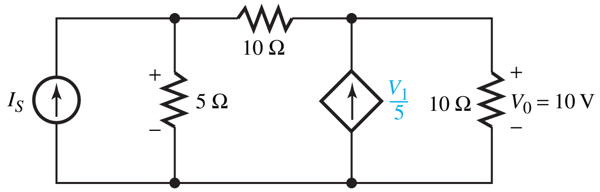

Figure P1.3.8

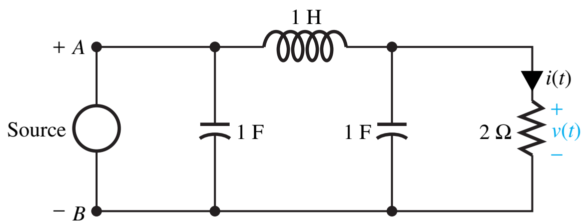

Figure P1.3.9

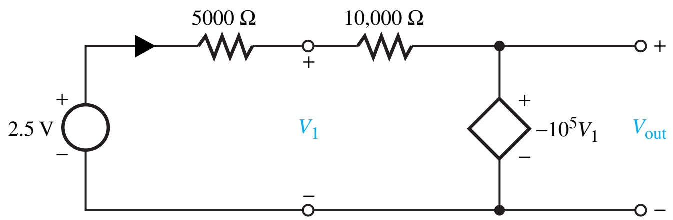

Figure P1.3.10

degree Celsius. If the temperature at the time of measurement is 20°C above ambient, compare the percent error of both meters when measuring a current of 5 A.

1.4.4 A DMM (digital multimeter) reads true rms values of current. If the peak value of each of the following periodic current waves is 5 A, find the meter reading for: (a) a sine wave, (b) a square wave, (c) a triangular wave, and (d) a sawtooth wave.

1.4.5 Consider the bridge circuit given in Figure P1.4.5 with $R_1 = 24\,\text{k}\Omega$, $R_2 = 48\,\text{k}\Omega$, and $R_3 = 10\,\text{k}\Omega$. Find R_4 when the bridge is balanced with $V_1 = 0$.

***1.4.6** In the Wheatstone bridge circuit shown in Figure P1.4.6, $R_1 = 16\,\Omega$, $R_2 = 8\,\Omega$, and $R_3 = 40\,\Omega$; R_4 is the unknown resistance. R_M is the galvanometer resistance of 6 Ω. If no current is detected by the galvanometer, when a 24-V source with a 12-Ω internal resistance is applied across terminals a–b, find: (a) R_4, and (b) the current through R_4.

1.4.7 Three waveforms seen on an oscilloscope are shown in Figure P1.4.7. If the horizontal scale is set to 50 ms per division (500 ms for the entire screen width), and the vertical scale is set to 5 mV per division (±25 mV for the entire screen height with zero voltage at the center), determine: (i) the maximum value of the voltage, and (ii) the frequency for each of the waveforms.

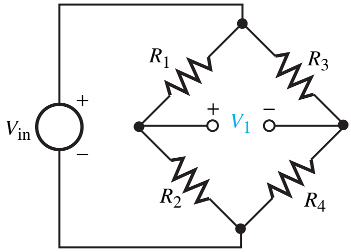

Figure P1.4.5

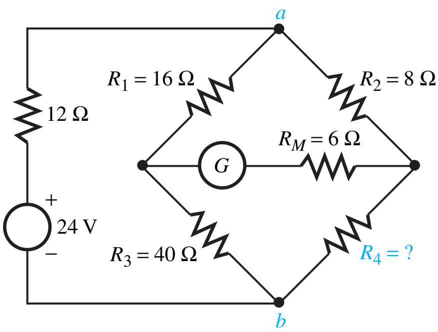

Figure P1.4.6

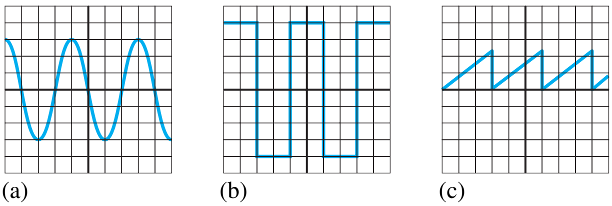

Figure P1.4.7 **(a)** Sinusoidal wave. **(b)** Rectangular wave. **(c)** Sawtooth wave.