## The Operational Amplifier

## KEY CONCEPTS

## INTRODUCTION

At this point we have a good set of circuit analysis tools at our disposal, but have focused primarily on somewhat general circuits composed of only sources and resistors. In this chapter, we introduce a new component which, although technically nonlinear, can be treated effectively with linear models. This element, known as the operational amplifier or op amp for short, finds daily usage in a large variety of electronic applications. It also provides us a new element to use in building circuits, and another opportunity to test out our developing analytical skills.

### 6.1 BACKGROUND

The origins of the operational amplifier date to the 1940s, when basic circuits were constructed using vacuum tubes to perform mathematical operations such as addition, subtraction, multiplication, division, differentiation, and integration. This enabled the construction of analog (as opposed to digital) computers tasked with the solution of complex differential equations. The first commercially available op amp device is generally considered to be the K2-W, manufactured by Philbrick Researches, Inc. of Boston from about 1952 through the early 1970s (Fig. 6.1a). These early vacuum tube devices weighed $3 \mathrm{oz}(85 \mathrm{~g})$, measured $133 / 64$ in $\times 2^{9} / 64$ in $\times$ $47 / 64$ in ( $3.8 \mathrm{~cm} \times 5.4 \mathrm{~cm} \times 10.4 \mathrm{~cm}$ ), and sold for about US\$22. In contrast, integrated circuit (IC) op amps such as the Fairchild KA741 weigh less than 500 mg , measure $5.7 \mathrm{~mm} \times 4.9 \mathrm{~mm} \times$ 1.8 mm , and sell for approximately US $\$ 0.22$.

Compared to op amps based on vacuum tubes, modern IC op amps are constructed using perhaps 25 or more transistors all on the same silicon "chip," as well as resistors and capacitors needed to obtain the desired performance characteristics. As a result, they run at

Characteristics of Ideal Op Amps

Inverting and Noninverting Amplifiers

Summing and Difference
Amplifier Circuits

Cascaded Op Amp Stages

Using Op Amps to Build
Voltage and Current Sources

Nonideal Characteristics of Op Amps

Voltage Gain and Feedback

Basic Comparator and Instrumentation Amplifier Circuits

(a)

(b)

FIGURE 6.2 (a) Electrical symbol for the op amp. (b) Minimum required connections to be shown on a circuit schematic.


■ FIGURE 6.1 (a) A Philbrick K2-W op amp, based on a matched pair of 12AX7A vacuum tubes. (b) LMV321 op amp, used in a variety of phone and game applications. (c) LMC6035 operational amplifier, which packs 114 transistors into a package so small that it fits on the head of a pin. (b-c) Copyright © 2011 National Semiconductor Corporation (www.national.com). All rights reserved. Used with permission.
much lower dc supply voltages ( $\pm 18 \mathrm{~V}$, for example, as opposed to $\pm 300 \mathrm{~V}$ for the K2-W), are more reliable, and considerably smaller (Fig. 6.1b,c). In some cases, the IC may contain several op amps. In addition to the output pin and the two inputs, other pins enable power to be supplied to run the transistors, and for external adjustments to be made to balance and compensate the op amp. The symbol commonly used for an op amp is shown in Fig. 6.2a. At this point, we are not concerned with the internal circuitry of the op amp or the IC, but only with the voltage and current relationships that exist between the input and output terminals. Thus, for the time being we will use a simpler electrical symbol, shown in Fig. 6.2b. Two input terminals are shown on the left, and a single output terminal appears at the right. The terminal marked by a " + " is referred to as the noninverting input, and the " - " marked terminal is called the inverting input.

### 6.2 THE IDEAL OP AMP: A CORDIAL INTRODUCTION

In practice, we find that most op amps perform so well that we can often make the assumption that we are dealing with an "ideal" op amp. The characteristics of an ideal op amp form the basis for two fundamental rules that at first may seem somewhat unusual:

## Ideal Op Amp Rules

1. No current ever flows into either input terminal.
2. There is no voltage difference between the two input terminals.

In a real op amp, a very small leakage current will flow into the input (sometimes as low as 40 femtoamperes). It is also possible to obtain a very small voltage across the two input terminals. However, compared to other voltages and currents in most circuits, such values are so small that including them in the analysis does not typically affect our calculations.

When analyzing op amp circuits, we should keep one other point in mind. As opposed to the circuits that we have studied so far, an op amp circuit always has an output that depends on some type of input. Therefore, we will analyze op amp circuits with the goal of obtaining an expression for the output in terms of the input quantities. We will find that it is usually a good idea to begin the analysis of an op amp circuit at the input, and proceed from there.


The circuit shown in Fig. 6.3 is known as an inverting amplifier. We choose to analyze this circuit using KVL, beginning with the input voltage source. The current labeled $i$ flows only through the two resistors $R_{1}$ and $R_{f}$; ideal op amp rule 1 states that no current flows into the inverting input terminal. Thus, we can write

$$
-v_{\mathrm{in}}+R_{1} i+R_{f} i+v_{\mathrm{out}}=0
$$

which can be rearranged to obtain an equation that relates the output to the input:

$$
\begin{equation*}
v_{\mathrm{out}}=v_{\mathrm{in}}-\left(R_{1}+R_{f}\right) i \tag{1}
\end{equation*}
$$

Given $v_{\text {in }}=5 \sin 3 t \mathrm{mV}, R_{1}=4.7 \mathrm{k} \Omega$, and $R_{f}=47 \mathrm{k} \Omega$, we require one additional equation that expresses $i$ only in terms of $v_{\text {out }}, v_{\mathrm{in}}, R_{1}$, and/or $R_{f}$.

This is a good time to mention that we have not yet made use of ideal op amp rule 2. Since the noninverting input is grounded, it is at zero volts. By ideal op amp rule 2, the inverting input is therefore also at zero volts! This does not mean that the two inputs are physically shorted together, and we should be careful not to make such an assumption. Rather, the two input voltages simply track each other: if we try to change the voltage at one pin, the other pin will be driven by internal circuitry to the same value. Thus, we can write one more KVL equation:

$$
-v_{\mathrm{in}}+R_{1} i+0=0
$$

or

$$
\begin{equation*}
i=\frac{v_{\text {in }}}{R_{1}} \tag{2}
\end{equation*}
$$

Combining Eq. [2] with Eq. [1], we obtain an expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$ :

$$
\begin{equation*}
v_{\mathrm{out}}=-\frac{R_{f}}{R_{1}} v_{\mathrm{in}} \tag{3}
\end{equation*}
$$

Substituting $v_{\text {in }}=5 \sin 3 t \mathrm{mV}, R_{1}=4.7 \mathrm{k} \Omega$, and $R_{f}=47 \mathrm{k} \Omega$,

$$
v_{\text {out }}=-50 \sin 3 t \quad \mathrm{mV}
$$

Since $R_{f}>R_{1}$, this circuit amplifies the input voltage signal $v_{\text {in }}$. If we choose $R_{f}<R_{1}$, the signal will be attenuated instead. We also note that the output voltage has the opposite sign of the input voltage, ${ }^{1}$ hence the name "inverting amplifier." The output is sketched in Fig. 6.4, along with the input waveform for comparison.

At this point, it is worth mentioning that the ideal op amp seems to be violating KCL. Specifically, in the above circuit no current flows into or out of either input terminal, but somehow current is able to flow into the output pin! This would imply that the op amp is somehow able to either create electrons out of nowhere or store them forever (depending on the direction of current flow). Obviously, this is not possible. The conflict arises because we have been treating the op amp the same way we treated passive elements


FIGURE 6.3 An op amp used to construct an inverting amplifier circuit. The current $i$ flows to ground through the output pin of the op amp.

The fact that the inverting input terminal finds itself at zero volts in this type of circuit configuration leads to what is often referred to as a "virtual ground." This does not mean that the pin is actually grounded, which is sometimes a source of confusion for students. The op amp makes whatever internal adjustments are necessary to prevent a voltage difference between the input terminals. The input terminals are not shorted together.


FIGURE 6.4 Input and output waveforms of the inverting amplifier circuit.


FIGURE 6.5 An inverting amplifier circuit with a 2.5 V input.
such as the resistor. In reality, however, the op amp cannot function unless it is connected to external power sources. It is through those power sources that we can direct current flow through the output terminal.

Although we have shown that the inverting amplifier circuit of Fig. 6.3 can amplify an ac signal (a sine wave in this case having a frequency of $3 \mathrm{rad} / \mathrm{s}$ and an amplitude of 5 mV ), it works just as well with dc inputs. We consider this type of situation in Fig. 6.5, where values for $R_{1}$ and $R_{f}$ are to be selected to obtain an output voltage of -10 V .

This is the same circuit as shown in Fig. 6.3, but with a 2.5 V dc input. Since no other change has been made, the expression we presented as Eq. [3] is valid for this circuit as well. To obtain the desired output, we seek a ratio of $R_{f}$ to $R_{1}$ of $10 / 2.5$, or 4 . Since it is only the ratio that is important here, we simply need to pick a convenient value for one resistor, and the other resistor value is then fixed at the same time. For example, we could choose $R_{1}=100 \Omega$ (so $R_{f}=400 \Omega$ ), or even $R_{f}=8 \mathrm{M} \Omega$ (so $R_{1}=2 \mathrm{M} \Omega$ ). In practice, other constraints (such as bias current) may limit our choices.

This circuit configuration therefore acts as a convenient type of voltage amplifier (or attenuator, if the ratio of $R_{f}$ to $R_{1}$ is less than 1 ), but does have the sometimes inconvenient property of inverting the sign of the input. There is an alternative, however, which is analyzed just as easily-the noninverting amplifier shown in Fig. 6.6. We examine such a circuit in the following example.

## EXAMPLE 6.1


(a)

(b)

FIGURE 6.6 (a) An op amp used to construct a noninverting amplifier circuit. (b) Circuit with the current through $R_{1}$ and $R_{f}$ defined, as well as both input voltages labeled.

Sketch the output waveform of the noninverting amplifier circuit in Fig. $6.6 a$. Use $v_{\text {in }}=5 \sin 3 t \mathrm{mV}, R_{1}=4.7 \mathrm{k} \Omega$, and $R_{f}=47 \mathrm{k} \Omega$.

## Identify the goal of the problem.

We require an expression for $v_{\text {out }}$ that only depends on the known quantities $v_{\mathrm{in}}, R_{1}$, and $R_{f}$.

## Collect the known information.

Since values have been specified for the resistors and the input waveform, we begin by labeling the current $i$ and the two input voltages as shown in Fig. 6.6b. We will assume that the op amp is an ideal op amp.

## Devise a plan.

Although mesh analysis is a favorite technique of students, it turns out to be more practical in most op amp circuits to apply nodal analysis, since there is no direct way to determine the current flowing out of the op amp output.

## Construct an appropriate set of equations.

Note that we are using ideal op amp rule 1 implicitly by defining the same current through both resistors: no current flows into the inverting input terminal. Employing nodal analysis to obtain our expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$, we thus find that

At node $a$ :

$$
\begin{equation*}
0=\frac{v_{a}}{R_{1}}+\frac{v_{a}-v_{\text {out }}}{R_{f}} \tag{4}
\end{equation*}
$$

At node $b$ :

$$
\begin{equation*}
v_{b}=v_{\text {in }} \tag{5}
\end{equation*}
$$

## Determine if additional information is required.

Our goal is to obtain a single expression that relates the input and output voltages, although neither Eq. [4] nor Eq. [5] appears to do so. However, we have not yet employed ideal op amp rule 2, and we will find that in almost every op amp circuit both rules need to be invoked in order to obtain such an expression.

Thus, we recognize that $v_{a}=v_{b}=v_{\text {in }}$, and Eq. [4] becomes

$$
0=\frac{v_{\text {in }}}{R_{1}}+\frac{v_{\text {in }}-v_{\text {out }}}{R_{f}}
$$

## Attempt a solution.

Rearranging, we obtain an expression for the output voltage in terms of the input voltage $v_{\text {in }}$ :

$$
v_{\mathrm{out}}=\left(1+\frac{R_{f}}{R_{1}}\right) v_{\mathrm{in}}=11 v_{\mathrm{in}}=55 \sin 3 t \mathrm{mV}
$$

## Verify the solution. Is it reasonable or expected?

The output waveform is sketched in Fig. 6.7, along with the input waveform for comparison. In contrast to the output waveform of the inverting amplifier circuit, we note that the input and output are in phase for the noninverting amplifier. This should not be entirely unexpected: it is implicit in the name "noninverting amplifier."

## PRACTICE

6.1 Derive an expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$ for the circuit shown in Fig. 6.8.

Ans: $v_{\text {out }}=v_{\text {in }}$. The circuit is known as a "voltage follower," since the output voltage tracks or "follows" the input voltage.


FIGURE 6.7 Input and output waveforms for the noninverting amplifier circuit.


Just like the inverting amplifier, the noninverting amplifier works with dc as well as ac inputs, but has a voltage gain of $v_{\text {out }} / v_{\text {in }}=1+\left(R_{f} / R_{1}\right)$. Thus, if we set $R_{f}=9 \Omega$ and $R_{1}=1 \Omega$, we obtain an output $v_{\text {out }}$ which is 10 times larger than the input voltage $v_{\text {in }}$. In contrast to the inverting amplifier, the output and input of the noninverting amplifier always have the same sign, and the output voltage cannot be less than the input; the minimum gain is 1 . Which amplifier we choose depends on the application we are considering. In the special case of the voltage follower circuit shown in Fig. 6.8,
which represents a noninverting amplifier with $R_{1}$ set to $\infty$ and $R_{f}$ set to zero, the output is identical to the input in both sign and magnitude. This may seem rather pointless as a general type of circuit, but we should keep in mind that the voltage follower draws no current from the input (in the ideal case)-it therefore can act as a buffer between the voltage $v_{\text {in }}$ and some resistive load $R_{L}$ connected to the output of the op amp.

We mentioned earlier that the name "operational amplifier" originates from using such devices to perform arithmetical operations on analog (i.e., nondigitized, real-time, real-world) signals. As we see in the following two circuits, this includes both addition and subtraction of input voltage signals.

## EXAMPLE 6.2

Obtain an expression for $v_{\text {out }}$ in terms of $v_{1}, v_{2}$, and $v_{3}$ for the op amp circuit in Fig. 6.9, also known as a summing amplifier.


FIGURE 6.9 Basic summing amplifier circuit with three inputs.
We first note that this circuit is similar to the inverting amplifier circuit of Fig. 6.3. Again, the goal is to obtain an expression for $v_{\text {out }}$ (which in this case appears across a load resistor $R_{L}$ ) in terms of the inputs $\left(v_{1}, v_{2}\right.$, and $\left.v_{3}\right)$.

Since no current can flow into the inverting input terminal, we can write

$$
i=i_{1}+i_{2}+i_{3}
$$

Therefore, we can write the following equation at the node labeled $v_{a}$ :

$$
0=\frac{v_{a}-v_{\text {out }}}{R_{f}}+\frac{v_{a}-v_{1}}{R}+\frac{v_{a}-v_{2}}{R}+\frac{v_{a}-v_{3}}{R}
$$

This equation contains both $v_{\text {out }}$ and the input voltages, but unfortunately it also contains the nodal voltage $v_{a}$. To remove this unknown quantity from our expression, we need to write an additional equation that relates $v_{a}$ to $v_{\text {out }}$, the input voltages, $R_{f}$, and/or $R$. At this point, we remember that we have not yet used ideal op amp rule 2 , and that we will almost certainly require the use of both rules when analyzing an op amp circuit. Thus, since $v_{a}=v_{b}=0$, we can write the following:

$$
0=\frac{v_{\text {out }}}{R_{f}}+\frac{v_{1}}{R}+\frac{v_{2}}{R}+\frac{v_{3}}{R}
$$

Rearranging, we obtain the following expression for $v_{\text {out }}$ :

$$
\begin{equation*}
v_{\mathrm{out}}=-\frac{R_{f}}{R}\left(v_{1}+v_{2}+v_{3}\right) \tag{6}
\end{equation*}
$$

In the special case where $v_{2}=v_{3}=0$, we see that our result agrees with Eq. [3], which was derived for essentially the same circuit.

There are several interesting features about the result we have just derived. First, if we select $R_{f}$ so that it is equal to $R$, then the output is the (negative of the) sum of the three input signals $v_{1}, v_{2}$, and $v_{3}$. Further, we can select the ratio of $R_{f}$ to $R$ to multiply this sum by a fixed constant. So, for example, if the three voltages represented signals from three separate scales calibrated so that $-1 \mathrm{~V}=1 \mathrm{lb}$, we could set $R_{f}=R / 2.205$ to obtain a voltage signal that represented the combined weight in kilograms (to within about 1 percent accuracy due to our conversion factor).

Also, we notice that $R_{L}$ did not appear in our final expression. As long as its value is not too low, the operation of the circuit will not be affected; at present, we have not considered a detailed enough model of an op amp to predict such an occurrence. This resistor represents the Thévenin equivalent of whatever we use to monitor the amplifier output. If our output device is a simple voltmeter, then $R_{L}$ represents the Thévenin equivalent resistance seen looking into the voltmeter terminals (typically $10 \mathrm{M} \Omega$ or more). Or, our output device might be a speaker (typically $8 \Omega$ ), in which case we hear the sum of the three separate sources of sound; $v_{1}, v_{2}$, and $v_{3}$ might represent microphones in that case.

One word of caution: It is frequently tempting to assume that the current labeled $i$ in Fig. 6.9 flows not only through $R_{f}$ but through $R_{L}$ also. Not true! It is very possible that current is flowing through the output terminal of the op amp as well, so that the currents through the two resistors are not the same. It is for this reason that we almost universally avoid writing KCL equations at the output pin of an op amp, which leads to the preference of nodal over mesh analysis when working with most op amp circuits.

For convenience, we summarize the most common op amp circuits in Table 1.

## PRACTICE

6.2 Derive an expression for $v_{\text {out }}$ in terms of $v_{1}$ and $v_{2}$ for the circuit shown in Fig. 6.10, also known as a difference amplifier.


## FIGURE 6.10

[^0]

## TABLE 6.1 Summary of Basic Op Amp Circuits

## Name

Inverting Amplifier


Noninverting Amplifier


Voltage Follower
(also known as a Unity Gain Amplifier)

$v_{\text {out }}=\left(1+\frac{R_{f}}{R_{1}}\right) v_{\text {in }}$

$$
v_{\text {out }}=v_{\text {in }}
$$

$$
v_{\mathrm{out}}=-\frac{R_{f}}{R}\left(v_{1}+v_{2}+v_{3}\right)
$$

## PRACTICAL APPLICATION

## A Fiber Optic Intercom

A point-to-point intercom system can be constructed using a number of different approaches, depending on the intended application environment. Low-power radio frequency (RF) systems work very well and are generally cost-effective, but are subject to interference from other RF sources and are also prone to eavesdropping. Use of a simple wire to connect the two intercom systems instead can eliminate a great deal of the RF interference as well as increase privacy. However, wires are subject to corrosion and short circuits when the plastic insulation wears, and their weight can be a concern in aircraft and related applications (Fig. 6.11).


FIGURE 6.11 The application environment often dictates design constraints. (@ Michael Melford/Riser/Getty Images.)

An alternative design would be to convert the electrical signal from the microphone to an optical signal, which could then be transmitted through a thin ( $\sim 50 \mu \mathrm{~m}$ diameter) optical fiber. The optical signal is then converted back to an electrical signal, which is amplified and delivered to a speaker. A schematic diagram of such a system is shown in Fig. 6.12; two such systems would be needed for two-way communication.


FIGURE 6.12 Schematic diagram of one-half of a simple fiber optic intercom.

We can consider the design of the transmission and reception circuits separately, since the two circuits are in fact electrically independent. Figure 6.13 shows a simple


FIGURE 6.13 Circuit used to convert the electrical microphone signal into an optical signal for transmission through a fiber.
signal generation circuit consisting of a microphone, a light-emitting diode (LED), and an op amp used in a noninverting amplifier circuit to drive the LED; not shown are the power connections required for the op amp itself. The light output of the LED is roughly proportional to its current, although less so for very small and very large values of current.

We know the gain of the amplifier is given by

$$
\frac{v_{\text {out }}}{v_{\text {in }}}=1+\frac{R_{f}}{R_{1}}
$$

which is independent of the resistance of the LED. In order to select values for $R_{f}$ and $R_{1}$, we need to know the input voltage from the microphone and the necessary output voltage to power the LED. A quick measurement indicates that the typical voltage output of the microphone peaks at 40 mV when someone is using a normal speaking voice. The LED manufacturer recommends operating at approximately 1.6 V , so we design for a gain of $1.6 / 0.04=40$. Arbitrarily choosing $R_{1}=1 \mathrm{k} \Omega$ leads to a required value of $39 \mathrm{k} \Omega$ for $R_{f}$.

The circuit of Fig. 6.14 is the receiver part of our oneway intercom system. It converts the optical signal from the fiber into an electrical signal, amplifying it so that an audible sound emanates from the speaker.


FIGURE 6.14 Receiver circuit used to convert the optical signal into an audio signal.

After coupling the LED output of the transmitting circuit to the optical fiber, a signal of approximately 10 mV is measured from the photodetector. The speaker is rated for a maximum of 100 mW and has an equivalent resistance of $8 \Omega$. This equates to a maximum speaker voltage of 894 mV , so we need to select values of $R_{2}$ and $R_{3}$ to obtain a gain of $894 / 10=89.4$. With the arbitrary
selection of $R_{2}=10 \mathrm{k} \Omega$, we find that a value of $884 \mathrm{k} \Omega$ completes our design.

This circuit will work in practice, although the nonlinear characteristics of the LED lead to a noticeable distortion of the audio signal. We leave improved designs for more advanced texts.

### 6.3. CASCADED STAGES

Although the op amp is an extremely versatile device, there are numerous applications in which a single op amp will not suffice. In such instances, it is often possible to meet application requirements by cascading several individual op amps together in the same circuit. An example of this is shown in Fig. 6.15, which consists of the summing amplifier circuit of Fig. 6.9 with only two input sources, and the output fed into a simple inverting amplifier. The result is a two-stage op amp circuit.


■ FIGURE 6.15 A two-stage op amp circuit consisting of a summing amplifier cascaded with an inverting amplifier circuit.

We have already analyzed each of these op amp circuits separately. Based on our previous experience, if the two op amp circuits were disconnected, we would expect

$$
\begin{equation*}
v_{x}=-\frac{R_{f}}{R}\left(v_{1}+v_{2}\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{\text {out }}=-\frac{R_{2}}{R_{1}} v_{x} \tag{8}
\end{equation*}
$$

In fact, since the two circuits are connected at a single point and the voltage $v_{x}$ is not influenced by the connection, we can combine Eqs. [7] and [8] to obtain

$$
\begin{equation*}
v_{\mathrm{out}}=\frac{R_{2}}{R_{1}} \frac{R_{f}}{R}\left(v_{1}+v_{2}\right) \tag{9}
\end{equation*}
$$

which describes the input-output characteristics of the circuit shown in Fig. 6.15. We may not always be able to reduce such a circuit to familiar stages, however, so it is worth seeing how the two-stage circuit of Fig. 6.15 can be analyzed as a whole.

When analyzing cascaded circuits, it is sometimes helpful to begin with the last stage and work backward toward the input stage. Referring to ideal op amp rule 1 , the same current flows through $R_{1}$ and $R_{2}$. Writing the appropriate nodal equation at the node labeled $v_{c}$ yields

$$
\begin{equation*}
0=\frac{v_{c}-v_{x}}{R_{1}}+\frac{v_{c}-v_{\text {out }}}{R_{2}} \tag{10}
\end{equation*}
$$

Applying ideal op amp rule 2, we can set $v_{c}=0$ in Eq. [10], resulting in

$$
\begin{equation*}
0=\frac{v_{x}}{R_{1}}+\frac{v_{\text {out }}}{R_{2}} \tag{11}
\end{equation*}
$$

Since our goal is an expression for $v_{\text {out }}$ in terms of $v_{1}$ and $v_{2}$, we proceed to the first op amp in order to obtain an expression for $v_{x}$ in terms of the two input quantities.

Applying ideal op amp rule 1 at the inverting input of the first op amp,

$$
\begin{equation*}
0=\frac{v_{a}-v_{x}}{R_{f}}+\frac{v_{a}-v_{1}}{R}+\frac{v_{a}-v_{2}}{R} \tag{12}
\end{equation*}
$$

Ideal op amp rule 2 allows us to replace $v_{a}$ in Eq. [12] with zero, since $v_{a}=v_{b}=0$. Thus, Eq. [12] becomes

$$
\begin{equation*}
0=\frac{v_{x}}{R_{f}}+\frac{v_{1}}{R}+\frac{v_{2}}{R} \tag{13}
\end{equation*}
$$

We now have an equation for $v_{\text {out }}$ in terms of $v_{x}$ (Eq. [11]), and an equation for $v_{x}$ in terms of $v_{1}$ and $v_{2}$ (Eq. [13]). These equations are identical to Eqs. [7] and [8], respectively, which means that cascading the two separate circuits as in Fig. 6.15 did not affect the input-output relationship of either stage. Combining Eqs. [11] and [13], we find that the input-output relationship for the cascaded op amp circuit is

$$
\begin{equation*}
v_{\text {out }}=\frac{R_{2}}{R_{1}} \frac{R_{f}}{R}\left(v_{1}+v_{2}\right) \tag{14}
\end{equation*}
$$

which is identical to Eq. [9].
Thus, the cascaded circuit acts as a summing amplifier, but without a phase reversal between the input and output. By choosing the resistor values carefully, we can either amplify or attenuate the sum of the two input voltages. If we select $R_{2}=R_{1}$ and $R_{f}=R$, we can also obtain an amplifier circuit where $v_{\text {out }}=v_{1}+v_{2}$, if desired.

## EXAMPLE 6.3


#### Abstract

A multiple-tank gas propellant fuel system is installed in a small lunar orbit runabout. The amount of fuel in any tank is monitored by measuring the tank pressure (in psia). ${ }^{2}$ Technical details for tank capacity as well as sensor pressure and voltage range are given in Table 6.2. Design a circuit which provides a positive dc voltage signal proportional to the total fuel remaining, such that $1 \mathrm{~V}=100$ percent.



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## TABLE 6.2 Technical Data for Tank Pressure Monitoring System

We see from Table 6.2 that the system has three separate gas tanks, requiring three separate sensors. Each sensor is rated up to 12,500 psia, with a corresponding output of 5 V . Thus, when tank 1 is full, its sensor will provide a voltage signal of $5 \times(10,000 / 12,500)=4 \mathrm{~V}$; the same is true for the sensor monitoring tank 2 . The sensor connected to tank 3, however, will only provide a maximum voltage signal of $5 \times(2000 / 12,500)=800 \mathrm{mV}$.

One possible solution is the circuit shown in Fig. 6.16a, which employs a summing amplifier stage with $v_{1}, v_{2}$, and $v_{3}$ representing the three sensor outputs, followed by an inverting amplifier to adjust the voltage sign and magnitude. Since we are not told the output resistance of the sensor, we employ a buffer for each one as shown in Fig. 6.16b; the result is (in the ideal case) no current flow from the sensor.

To keep the design as simple as possible, we begin by choosing $R_{1}, R_{2}, R_{3}$, and $R_{4}$ to be $1 \mathrm{k} \Omega$; any value will do as long as all four resistors are equal. Thus, the output of the summing stage is

$$
v_{x}=-\left(v_{1}+v_{2}+v_{3}\right)
$$

The final stage must invert this voltage and scale it such that the output voltage is 1 V when all three tanks are full. The full condition results in $v_{x}=-(4+4+0.8)=-8.8 \mathrm{~V}$. Thus, the final stage needs a voltage ratio of $R_{6} / R_{5}=1 / 8.8$. Arbitrarily choosing $R_{6}=1 \mathrm{k} \Omega$, we find that a value of $8.8 \mathrm{k} \Omega$ for $R_{5}$ completes the design.

[^1]
(a)

(b)

■ FIGURE 6.16 (a) A proposed circuit to provide a total fuel remaining readout. (b) Buffer design to avoid errors associated with the internal resistance of the sensor and limitations on its ability to provide current. One such buffer is used for each sensor, providing the inputs $v_{1}, v_{2}$, and $v_{3}$ to the summing amplifier stage.

## PRACTICE

6.3 An historic bridge is showing signs of deterioration. Until renovations can be performed, it is decided that only cars weighing less than 1600 kg will be allowed across. To monitor this, a four-pad weighing system is designed. There are four independent voltage signals, one from each wheel pad, with $1 \mathrm{mV}=1 \mathrm{~kg}$. Design a circuit to provide a positive voltage signal to be displayed on a DMM (digital multimeter) that represents the total weight of a vehicle, such that $1 \mathrm{mV}=1 \mathrm{~kg}$. You may assume there is no need to buffer the wheel pad voltage signals.

## Ans: See Fig. 6.17.



■ FIGURE 6.17 One possible solution to Practice Problem 6.3; all resistors are $10 \mathrm{k} \Omega$ (although any value will do as long as they are all equal). Input voltages $v_{1}, v_{2}, v_{3}$, and $v_{4}$ represent the voltage signals from the four wheel pad sensors, and $v_{\text {out }}$ is the output signal to be connected to the positive input terminal of the DMM. All five voltages are referenced to ground, and the common terminal of the DMM should be connected to ground as well.

### 6.4. CIRCUITS FOR VOLTAGE AND CURRENT SOURCES

In this and previous chapters we have often made use of ideal current and voltage sources, which we assume provide the same value of current or voltage, respectively, regardless of how they are connected in a circuit. Our assumption of independence has its limits, of course, as mentioned in Sec. 5.2 when we discussed practical sources which included a "built-in" or inherent resistance. The effect of such a resistance was a reduction of the voltage output of a voltage source as more current was demanded, or a diminished current output as more voltage was required from a current source. As discussed in this section, it is possible to construct circuits with more reliable characteristics using op amps.

## A Reliable Voltage Source

One of the most common means of providing a stable and consistent reference voltage is to make use of a nonlinear device known as a Zener diode. Its symbol is a triangle with a Z-like line across the top of the triangle, as shown for a 1N750 in the circuit of Fig. 6.18a. Diodes are characterized by


(c)

FIGURE 6.18 (a) PSpice schematic of a simple voltage reference circuit based on the 1 N 750 Zener diode. (b) Simulation of the circuit showing the diode voltage $\mathrm{V}_{\text {ref }}$ as a function of the driving voltage V . (c) Simulation of the diode current, showing that its maximum rating is exceeded when V1 exceeds 12.3 V . (Note that performing this calculation assuming an ideal Zener diode yields 12.2 V .)
a strongly asymmetric current-voltage relationship. For small voltages, they either conduct essentially zero current-or experience an exponentially increasing current-depending on the voltage polarity. In this way, they distinguish themselves from the simple resistor, where the magnitude of the current is the same for either voltage polarity and hence the resistor currentvoltage relationship is symmetric. Consequently, the terminals of a diode are not interchangeable, and have unique names: the anode (the flat part of the triangle) and the cathode (the point of the triangle).

A Zener diode is a special type of diode designed to be used with a positive voltage at the cathode with respect to the anode; when connected this way, the diode is said to be reverse biased. For low voltages, the diode acts like a resistor with a small linear increase in current flow as the voltage is increased. Once a certain voltage $\left(V_{\mathrm{BR}}\right)$ is reached, however-known as the reverse breakdown voltage or Zener voltage of the diode-the voltage does not significantly increase further, but essentially any current can flow up to the maximum rating of the diode ( 75 mA for a 1 N 750 , whose Zener voltage is 4.7 V ).

Let's consider the simulation result presented in Fig. $6.18 b$, which shows the voltage $\mathrm{V}_{\text {ref }}$ across the diode as the voltage source V 1 is swept from 0 to 20 V. Provided V1 remains above 5 V, the voltage across our diode is essentially constant. Thus, we could replace V1 with a 9 V battery, and not be too concerned with changes in our voltage reference as the battery voltage begins to drop as it discharges. The purpose of R1 in this circuit is simply to provide the necessary voltage drop between the battery and the diode; its value should be chosen to ensure that the diode is operating at its Zener voltage but below its maximum rated current. For example, Fig. 6.18c shows that the 75 mA rating is exceeded in our circuit if the source voltage V1 is much greater than 12 V . Thus, the value of resistor R 1 should be sized corresponding to the source voltage available, as we explore in Example 6.4.

Design a circuit based on the 1 N 750 Zener diode that runs on a single 9 V battery and provides a reference voltage of 4.7 V .

The 1N750 has a maximum current rating of 75 mA , and a Zener voltage of 4.7 V . The voltage of a 9 V battery can vary slightly depending on its state of charge, but we neglect this for the present design.

A simple circuit such as the one shown in Fig. $6.19 a$ is adequate for our purposes; the only issue is determining a suitable value for the resistor $R_{\text {ref }}$.

If 4.7 V is dropped across the diode, then $9-4.7=4.3 \mathrm{~V}$ must be dropped across $R_{\text {ref. }}$. Thus,

$$
R_{\mathrm{ref}}=\frac{9-V_{\mathrm{ref}}}{I_{\mathrm{ref}}}=\frac{4.3}{I_{\mathrm{ref}}}
$$

We determine $R_{\text {ref }}$ by specifying a current value. We know that $I_{\text {ref }}$ should not be allowed to exceed 75 mA for this diode, and large currents will discharge the battery more quickly. However, as seen in Fig. 6.19b, we cannot simply select $I_{\text {ref }}$ arbitrarily; very low currents do not allow

(a)

(b)

(c)

FIGURE 6.19 (a) A voltage reference circuit based on the 1 N750 Zener diode. (b) Diode - -V relationship. (c) PSpice simulation of the final design.
the diode to operate in the Zener breakdown region. In the absence of a detailed equation for the diode's current-voltage relationship (which is clearly nonlinear), we design for 50 percent of the maximum rated current as a rule of thumb. Thus,

$$
R_{\mathrm{ref}}=\frac{4.3}{0.0375}=115 \Omega
$$

Detailed "tweaking" can be obtained by performing a PSpice simulation of the final circuit, although we see from Fig. 6.19c that our first pass is reasonably close (within 1 percent) to our target value.

The basic Zener diode voltage reference circuit of Fig. $6.18 a$ works very well in many situations, but we are limited somewhat in the value of the voltage depending on which Zener diodes are available. Also, we often find that the circuit shown is not well suited to applications requiring more than a few milliamperes of current. In such instances, we may use the Zener reference circuit in conjunction with a simple amplifier stage, as shown in Fig. 6.20. The result is a stable voltage that can be controlled by adjusting the value of either $R_{1}$ or $R_{f}$, without having to switch to a different Zener diode.


FIGURE 6.20 An op amp-based voltage source using on a Zener voltage reference.

## PRACTICE

6.4 Design a circuit to provide a reference voltage of 6 V using a 1N750 Zener diode and a noninverting amplifier.

Ans: Using the circuit topology shown in Fig. 6.20, choose $V_{\text {bat }}=9 \mathrm{~V}$,
$R_{\text {ref }}=115 \Omega, R_{1}=1 \mathrm{k} \Omega$, and $R_{f}=268 \Omega$.

## A Reliable Current Source

Consider the circuit shown in Fig. 6.21a, where $V_{\text {ref }}$ is provided by a regulated voltage source such as the one shown in Fig. 6.19a. The reader may recognize this circuit as a simple inverting amplifier configuration, assuming we tap the output pin of the op amp. We can also use this circuit as a current source, however, where $R_{L}$ represents a resistive load.

The input voltage $V_{\text {ref }}$ appears across reference resistor $R_{\text {ref }}$, since the noninverting input of the op amp is connected to ground. With no current


FIGURE 6.21 (a) An op amp-based current source, controlled by the reference voltage $V_{\text {ref }}$.
(b) Circuit redrawn to highlight load. (c) Circuit model. Resistor $R_{L}$ represents the Norton equivalent of an unknown passive load circuit.
flowing into the inverting input, the current flowing through the load resistor $R_{L}$ is simply

$$
I_{s}=\frac{V_{\mathrm{ref}}}{R_{\mathrm{ref}}}
$$

In other words, the current supplied to $R_{L}$ does not depend on its resistance-the primary attribute of an ideal current source. It is also worth noting that we are not tapping the output voltage of the op amp here as a quantity of interest. Instead, we may view the load resistor $R_{L}$ as the Norton (or Thévenin) equivalent of some unknown passive load circuit, which receives power from the op amp circuit. Redrawing the circuit slightly as in Fig. 6.21b, we see that it has a great deal in common with the more familiar circuit of Fig. 6.21c. In other words, we may use this op amp circuit as an independent current source with essentially ideal characteristics, up to the maximum rated output current of the op amp selected.

## Design a current source that will deliver 1 mA to an arbitrary resistive load.

Basing our design on the circuits of Fig. 6.20 and Fig. 6.21a, we know that the current through our load $R_{L}$ will be given by

$$
I_{s}=\frac{V_{\mathrm{ref}}}{R_{\mathrm{ref}}}
$$

where values for $V_{\text {ref }}$ and $R_{\text {ref }}$ must be selected, and a circuit to provide $V_{\text {ref }}$ must also be designed. If we use a 1 N 750 Zener diode in series with a 9 V


FIGURE 6.22 One possible design for the desired current source. Note the change in current direction from Fig. 6.21b.


FIGURE 6.24 A more detailed model for the op amp.
battery and a $100 \Omega$ resistor, we know from Fig. $6.18 b$ that a voltage of 4.9 V will exist across the diode. Thus, $V_{\text {ref }}=4.9 \mathrm{~V}$, dictating a value of $4.9 / 10^{-3}=4.9 \mathrm{k} \Omega$ for $R_{\text {ref. }}$. The complete circuit is shown in Fig. 6.22.

Note that if we had assumed a diode voltage of 4.7 V instead, the error in our designed current would only be a few percent, well within the typical 5 to 10 percent tolerance in resistor values we might expect.

The only issue remaining is whether 1 mA can in fact be provided to any value of $R_{L}$. For the case of $R_{L}=0$, the output of the op amp will be 4.9 V , which is not unreasonable. As the load resistor is increased, however, the op amp output voltage increases. Eventually we must reach some type of limit, as discussed in Sec. 6.5.

## PRACTICE

6.5 Design a current source capable of providing $500 \mu \mathrm{~A}$ to a resistive load.

Ans: See Fig. 6.23 for one possible solution.


■ FIGURE 6.23 One possible solution to Practice Problem 6.5.

## 6.5 . PRACTICAL CONSIDERATIONS

## A More Detailed Op Amp Model

Reduced to its essentials, the op amp can be thought of as a voltagecontrolled dependent voltage source. The dependent voltage source provides the output of the op amp, and the voltage on which it depends is applied to the input terminals. A schematic diagram of a reasonable model for a practical op amp is shown in Fig. 6.24; it includes a dependent voltage source with voltage gain $A$, an output resistance $R_{o}$, and an input resistance $R_{i}$. Table 6.3 gives typical values for these parameters for several types of commercially available op amps.

The parameter $A$ is referred to as the open-loop voltage gain of the op amp, and is typically in the range of $10^{5}$ to $10^{6}$. We notice that all of the op amps listed in Table 6.3 have extremely large open-loop voltage gain, especially compared to the voltage gain of 11 that characterized the noninverting amplifier circuit of Example 6.1. It is important to remember the distinction

## TABLE 6.3 Typical Parameter Values for Several Types of Op Amps

| Part Number | $\mu$ A741 | LM324 | LF411 | AD549K | OPA690 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Description | General purpose | Low-power quad | Low-offset, lowdrift JFET input | Ultralow input bias current | Wideband video frequency op amp |
| Open loop gain A | $2 \times 10^{5} \mathrm{~V} / \mathrm{V}$ | $10^{5} \mathrm{~V} / \mathrm{V}$ | $2 \times 10^{5} \mathrm{~V} / \mathrm{V}$ | $10^{6} \mathrm{~V} / \mathrm{V}$ | 2800 V/V |
| Input resistance | $2 \mathrm{M} \Omega$ | * | $1 \mathrm{~T} \Omega$ | $10 \mathrm{~T} \Omega$ | $190 \mathrm{k} \Omega$ |
| Output resistance | $75 \Omega$ | * | $\sim 1 \Omega$ | $\sim 15 \Omega$ | * |
| Input bias current | 80 nA | 45 nA | 50 pA | 75 fA | $3 \mu \mathrm{~A}$ |
| Input offset voltage | 1.0 mV | 2.0 mV | 0.8 mV | 0.150 mV | $\pm 1.0 \mathrm{mV}$ |
| CMRR | 90 dB | 85 dB | 100 dB | 100 dB | 65 dB |
| Slew rate | $0.5 \mathrm{~V} / \mu \mathrm{s}$ | * | $15 \mathrm{~V} / \mu \mathrm{s}$ | $3 \mathrm{~V} / \mu \mathrm{s}$ | $1800 \mathrm{~V} / \mu \mathrm{s}$ |
| PSpice Model | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |

between the open-loop voltage gain of the op amp itself, and the closedloop voltage gain that characterizes a particular op amp circuit. The "loop" in this case refers to an external path between the output pin and the inverting input pin; it can be a wire, a resistor, or another type of element, depending on the application.

The $\mu$ A741 is a very common op amp, originally produced by Fairchild Corporation in 1968. It is characterized by an open-loop voltage gain of 200,000 , an input resistance of $2 \mathrm{M} \Omega$, and an output resistance of $75 \Omega$. In order to evaluate how well the ideal op amp model approximates the behavior of this particular device, let's revisit the inverting amplifier circuit of Fig. 6.3.

Using the appropriate values for the $\mu \mathrm{A} 741 \mathrm{op} \mathrm{amp}$ in the model of Fig. 6.24, reanalyze the inverting amplifier circuit of Fig. 6.3.

We begin by replacing the ideal op amp symbol of Fig. 6.3 with the detailed model, resulting in the circuit shown in Fig. 6.25.
Note that we can no longer invoke the ideal op amp rules, since we are not using the ideal op amp model. Thus, we write two nodal equations:

$$
\begin{aligned}
& 0=\frac{-v_{d}-v_{\text {in }}}{R_{1}}+\frac{-v_{d}-v_{\text {out }}}{R_{f}}+\frac{-v_{d}}{R_{i}} \\
& 0=\frac{v_{\text {out }}+v_{d}}{R_{f}}+\frac{v_{\text {out }}-A v_{d}}{R_{o}}
\end{aligned}
$$

Performing some straightforward but rather lengthy algebra, we eliminate $v_{d}$ and combine these two equations to obtain the following


■ FIGURE 6.25 Inverting amplifier circuit drawn using detailed op amp model.
expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$ :

$$
\begin{equation*}
v_{\text {out }}=\left[\frac{R_{o}+R_{f}}{R_{o}-A R_{f}}\left(\frac{1}{R_{1}}+\frac{1}{R_{f}}+\frac{1}{R_{i}}\right)-\frac{1}{R_{f}}\right]^{-1} \frac{v_{\text {in }}}{R_{1}} \tag{15}
\end{equation*}
$$

Substituting $v_{\text {in }}=5 \sin 3 \mathrm{tmV}, R_{1}=4.7 \mathrm{k} \Omega, R_{f}=47 \mathrm{k} \Omega, R_{o}=75 \Omega$, $R_{i}=2 \mathrm{M} \Omega$, and $A=2 \times 10^{5}$, we obtain

$$
v_{\text {out }}=-9.999448 v_{\text {in }}=-49.99724 \sin 3 t \quad \mathrm{mV}
$$

Upon comparing this to the expression found assuming an ideal op $\operatorname{amp}\left(v_{\text {out }}=-10 v_{\text {in }}=-50 \sin 3 t \mathrm{mV}\right)$, we see that the ideal op amp is indeed a reasonably accurate model. Further, assuming an ideal op amp leads to a significant reduction in the algebra required to perform the circuit analysis. Note that if we allow $A \rightarrow \infty, R_{o} \rightarrow 0$, and $R_{i} \rightarrow \infty$, Eq. [15] reduces to

$$
v_{\mathrm{out}}=-\frac{R_{f}}{R_{1}} v_{\mathrm{in}}
$$

which is what we derived earlier for the inverting amplifier when assuming the op amp was ideal.

## PRACTICE

6.6 Assuming a finite open-loop gain $(A)$, a finite input resistance $\left(R_{i}\right)$, and zero output resistance $\left(R_{o}\right)$, derive an expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$ for the op amp circuit of Fig. 6.3.
Ans: $v_{\text {out }} / v_{\text {in }}=-A R_{f} R_{i} /\left[(1+A) R_{1} R_{i}+R_{1} R_{f}+R_{f} R_{i}\right]$.

## Derivation of the Ideal Op Amp Rules

We have seen that the ideal op amp can be a reasonably accurate model for the behavior of practical devices. However, using our more detailed model which includes a finite open-loop gain, finite input resistance, and nonzero output resistance, it is actually straightforward to derive the two ideal op amp rules.

Referring to Fig. 6.24, we see that the open circuit output voltage of a practical op amp can be expressed as

$$
\begin{equation*}
v_{\mathrm{out}}=A v_{d} \tag{16}
\end{equation*}
$$

Rearranging this equation, we find that $v_{d}$, sometimes referred to as the differential input voltage, can be written as

$$
\begin{equation*}
v_{d}=\frac{v_{\mathrm{out}}}{A} \tag{17}
\end{equation*}
$$

As we might expect, there are practical limits to the output voltage $v_{\text {out }}$ that can be obtained from a real op amp. As described in the next section, we must connect our op amp to external dc voltage supplies in order to power the internal circuitry. These external voltage supplies represent the maximum value of $v_{\text {out }}$, and are typically in the range of 5 to 24 V . If we divide 24 V by the open-loop gain of the $\mu \mathrm{A} 741\left(2 \times 10^{5}\right)$, we obtain $v_{d}=120 \mu \mathrm{~V}$. Although this is not the same as zero volts, such a small value compared to the output voltage of 24 V is practically zero. An ideal op amp would have infinite open-loop gain, resulting in $v_{d}=0$ regardless of $v_{\text {out }}$; this leads to ideal op amp rule 2.

Ideal op amp rule 1 states that "No current ever flows into either input terminal." Referring to Fig. 6.23, the input current of an op amp is simply

$$
i_{\text {in }}=\frac{v_{d}}{R_{i}}
$$

We have just determined that $v_{d}$ is typically a very small voltage. As we can see from Table 6.3, the input resistance of an op amp is very large, ranging from the megaohms to the teraohms! Using the value of $v_{d}=120 \mu \mathrm{~V}$ from above and $R_{i}=2 \mathrm{M} \Omega$, we compute an input current of 60 pA . This is an extremely small current, and we would require a specialized ammeter (known as a picoammeter) to measure it. We see from Table 6.3 that the typical input current (more accurately termed the input bias current) of a $\mu \mathrm{A} 741$ is 80 nA , three orders of magnitude larger than our estimate. This is a shortcoming of the op amp model we are using, which is not designed to provide accurate values for input bias current. Compared to the other currents flowing in a typical op amp circuit, however, either value is essentially zero. More modern op amps (such as the AD549) have even lower input bias currents. Thus, we conclude that ideal op amp rule 1 is a fairly reasonable assumption.

From our discussion, it is clear that an ideal op amp has infinite openloop voltage gain, and infinite input resistance. However, we have not yet considered the output resistance of the op amp and its possible effects on our circuit. Referring to Fig. 6.24, we see that

$$
v_{\mathrm{out}}=A v_{d}-R_{o} i_{\mathrm{out}}
$$

where $i_{\text {out }}$ flows from the output pin of the op amp. Thus, a nonzero value of $R_{o}$ acts to reduce the output voltage, an effect which becomes more pronounced as the output current increases. For this reason, an ideal op amp has an output resistance of zero ohms. The $\mu \mathrm{A} 741$ has a maximum output resistance of $75 \Omega$, and more modern devices such as the AD549 have even lower output resistance.

## Common-Mode Rejection

The op amp is occasionally referred to as a difference amplifier, since the output is proportional to the voltage difference between the two input


FIGURE 6.26 An op amp connected as a difference amplifier.
terminals. This means that if we apply identical voltages to both input terminals, we expect the output voltage to be zero. This ability of the op amp is one of its most attractive qualities, and is known as common-mode rejection. The circuit shown in Fig. 6.26 is connected to provide an output voltage

$$
v_{\mathrm{out}}=v_{2}-v_{1}
$$

If $v_{1}=2+3 \sin 3 t$ volts and $v_{2}=2$ volts, we would expect the output to be $-3 \sin 3 t$ volts; the 2 V component common to $v_{1}$ and $v_{2}$ would not be amplified, nor does it appear in the output.

For practical op amps, we do in fact find a small contribution to the output in response to common-mode signals. In order to compare one op amp type to another, it is often helpful to express the ability of an op amp to reject common-mode signals through a parameter known as the common-mode rejection ratio, or CMRR. Defining $v_{\mathrm{OCM}}$ as the output obtained when both inputs are equal $\left(v_{1}=v_{2}=v_{\mathrm{CM}}\right)$, we can determine $A_{\mathrm{CM}}$, the commonmode gain of the op amp

$$
A_{\mathrm{CM}}=\left|\frac{v_{\mathrm{o}_{\mathrm{CM}}}}{v_{\mathrm{CM}}}\right|
$$

We then define CMRR in terms of the ratio of differential-mode gain $A$ to the common-mode gain $A_{\mathrm{CM}}$, or

$$
\begin{equation*}
\mathrm{CMRR} \equiv\left|\frac{A}{A_{\mathrm{CM}}}\right| \tag{18}
\end{equation*}
$$

although this is often expressed in decibels (dB), a logarithmic scale:

$$
\begin{equation*}
\mathrm{CMRR}_{(\mathrm{dB})} \equiv 20 \log _{10}\left|\frac{A}{A_{\mathrm{CM}}}\right| \quad \mathrm{dB} \tag{19}
\end{equation*}
$$

Typical values for several different op amps are provided in Table 6.3; a value of 100 dB corresponds to an absolute ratio of $10^{5}$ for $A$ to $A_{\mathrm{CM}}$.

## Negative Feedback

We have seen that the open-loop gain of an op amp is very large, ideally infinite. In practical situations, however, its exact value can vary from the value specified by the manufacturer as typical. Temperature, for example, can have a number of significant effects on the performance of an op amp, so that the operating behavior in $-20^{\circ} \mathrm{C}$ weather may be significantly different from the behavior observed on a warm sunny day. Also, there are typically small variations between devices fabricated at different times. If we design a circuit in which the output voltage is the open-loop gain times the voltage at one of the input terminals, the output voltage could therefore be difficult to predict with a reasonable degree of precision, and might be expected to change depending on the ambient temperature.

A solution to such potential problems is to employ the technique of negative feedback, which is the process of subtracting a small portion of the output from the input. If some event changes the characteristics of the amplifier such that the output tries to increase, the input is decreasing at the same time. Too much negative feedback will prevent any useful amplification, but a small amount provides stability. An example of negative
feedback is the unpleasant sensation we feel as our hand draws near a flame. The closer we move toward the flame, the larger the negative signal sent from our hand. Overdoing the proportion of negative feedback, however, might cause us to abhor heat, and eventually freeze to death. Positive feedback is the process where some fraction of the output signal is added back to the input. A common example is when a microphone is directed toward a speaker-a very soft sound is rapidly amplified over and over until the system "screams." Positive feedback generally leads to an unstable system.

All of the circuits considered in this chapter incorporate negative feedback through the presence of a resistor between the output pin and the inverting input. The resulting loop between the output and the input reduces the dependency of the output voltage on the actual value of the open-loop gain (as seen in Example 6.6). This obviates the need to measure the precise open-loop gain of each op amp we use, as small variations in $A$ will not significantly impact the operation of the circuit. Negative feedback also provides increased stability in situations where $A$ is sensitive to the op amp's surroundings. For example, if $A$ suddenly increases in response to a change in the ambient temperature, a larger feedback voltage is added to the inverting input. This acts to reduce the differential input voltage $v_{d}$, and therefore the change in output voltage $A v_{d}$ is smaller. We should note that the closed-loop circuit gain is always less than the open-loop device gain; this is the price we pay for stability and reduced sensitivity to parameter variations.

## Saturation

So far, we have treated the op amp as a purely linear device, assuming that its characteristics are independent of the way in which it is connected in a circuit. In reality, it is necessary to supply power to an op amp in order to run the internal circuitry, as shown in Fig. 6.27. A positive supply, typically in the range of 5 to 24 V dc , is connected to the terminal marked $V^{+}$, and a negative supply of equal magnitude is connected to the terminal marked $V^{-}$. There are also a number of applications where a single voltage supply is acceptable, as well as situations where the two voltage magnitudes may be unequal. The op amp manufacturer will usually specify a maximum power supply voltage, beyond which damage to the internal transistors will occur.

The power supply voltages are a critical choice when designing an op amp circuit, because they represent the maximum possible output voltage of the op amp. ${ }^{3}$ For example, consider the op amp circuit shown in Fig. 6.26, now connected as a noninverting amplifier having a gain of 10 . As shown in the PSpice simulation in Fig. 6.28, we do in fact observe linear behavior from the op amp, but only in the range of $\pm 1.71 \mathrm{~V}$ for the input voltage. Outside of this range, the output voltage is no longer proportional to the input, reaching a peak magnitude of 17.6 V . This important nonlinear effect is known as saturation, which refers to the fact that further increases in the input voltage do not result in a change in the output voltage. This phenomenon refers to the fact that the output of a real op amp cannot exceed its


FIGURE 6.27 Op amp with positive and negative voltage supplies connected. Two 18 V supplies are used as an example; note the polarity of each source.


■ FIGURE 6.28 Simulated input-output characteristics of a $\mu \mathrm{A} 741$ connected as a noninverting amplifier with a gain of 10 , and powered by $\pm 18 \mathrm{~V}$ supplies.
supply voltages. For example, if we choose to run the op amp with a +9 V supply and a -5 V supply, then our output voltage will be limited to the range of -5 to +9 V . The output of the op amp is a linear response bounded by the positive and negative saturation regions, and as a general rule, we try to design our op amp circuits so that we do not accidentally enter the saturation region. This requires us to select the operating voltage carefully based on the closed-loop gain and maximum expected input voltage.

## Input Offset Voltage

As we are discovering, there are a number of practical considerations to keep in mind when working with op amps. One particular nonideality worth mentioning is the tendency for real op amps to have a nonzero output even when the two input terminals are shorted together. The value of the output under such conditions is known as the offset voltage, and the input voltage required to reduce the output to zero is referred to as the input offset voltage. Referring to Table 6.3, we see that typical values for the input offset voltage are on the order of a few millivolts or less.

Most op amps are provided with two pins marked either "offset null" or "balance." These terminals can be used to adjust the output voltage by connecting them to a variable resistor. A variable resistor is a three-terminal device commonly used for such applications as volume controls on radios. The device comes with a knob that can be rotated to select the actual value of resistance, and has three terminals. Measured between the two extreme terminals, its resistance is fixed regardless of the position of the knob. Using the middle terminal and one of the end terminals creates a resistor whose value depends on the knob position. Figure 6.29 shows a typical circuit used to adjust the output voltage of an op amp; the manufacturer's data sheet may suggest alternative circuitry for a particular device.

FIGURE 6.29 Suggested external circuitry for obtaining a zero output voltage. The $\pm 10 \mathrm{~V}$ supplies are shown as an example; the actual supply voltages used in the final circuit would be chosen in practice.

## Slew Rate

Up to now, we have tacitly assumed that the op amp will respond equally well to signals of any frequency, although perhaps we would not be surprised to find that in practice there is some type of limitation in this regard. Since we know that op amp circuits work well at dc, which is essentially zero frequency, it is the performance as the signal frequency is increased that we must consider. One measure of the frequency performance of an op amp is its slew rate, which is the rate at which the output voltage can respond to changes in the input; it is most often expressed in $\mathrm{V} / \mu \mathrm{s}$. The typical slew rate specification for several commercially available devices is provided in Table 6.3, showing values on the order of a few volts per microsecond. One notable exception is the OPA690, which is designed as a high-speed op amp for video applications requiring operation at several hundred MHz. As can be seen, a respectable slew rate of $1800 \mathrm{~V} / \mu \mathrm{s}$ is not unrealistic for this device, although its other parameters, particularly input bias current and CMRR, suffer somewhat as a result.

The PSpice simulations shown in Fig. 6.30 illustrate the degradation in performance of an op amp due to slew rate limitations. The circuit simulated is an LF411 configured as a noninverting amplifier with a gain of 2 and powered by $\pm 15 \mathrm{~V}$ supplies. The input waveform is shown in green, and has


FIGURE 6.30 Simulated performance of an LF411 op amp connected as a noninverting amplifier having a gain of 2 , with $\pm 15 \mathrm{~V}$ supplies and a pulsed input waveform. (a) Rise and fall times $=1 \mu \mathrm{~s}$, pulse width $=5 \mu \mathrm{~s}$; (b) rise and fall times $=100 \mathrm{~ns}$, pulse width $=500 \mathrm{~ns}$; (c) rise and fall times $=50 \mathrm{~ns}$, pulse width $=250 \mathrm{~ns}$.
a peak voltage of 1 V ; the output voltage is shown in red. The simulation of Fig. $6.30 a$ corresponds to a rise and fall time of $1 \mu \mathrm{~s}$ which, although a short time span for humans, is easily coped with by the LF411. As the rise and fall times are decreased by a factor of 10 to 100 ns (Fig. 6.30b), we begin to see that the LF411 is having a small difficulty in tracking the input. In the case of a 50 ns rise and fall time (Fig. 6.30c), we see that not only is there a significant delay between the output and the input, but the waveform is noticeably distorted as well-not a good feature of an amplifier. This observed behavior is consistent with the typical slew rate of $15 \mathrm{~V} / \mu$ s specified in Table 6.3, which indicates that the output might be expected to require roughly 130 ns to change from 0 to 2 V (or 2 V to 0 V ).

## Packaging

Modern op amps are available in a number of different types of packages. Some styles are better suited to high temperatures, and there are a variety of different ways to mount ICs on printed-circuit boards. Figure 6.31 shows several different styles of the LM741, manufactured by National Semiconductor. The label "NC" next to a pin means "no connection." The package styles shown in the figure are standard configurations, and are used for a large number of different integrated circuits; occasionally there are more pins available on a package than required.


FIGURE 6.31 Several different package styles for the LM741 op amp: (a) metal can; (b) dual-in-line package; (c) ceramic flatpak. (Copyright © 2011 National Semiconductor Corporation (ww.national.com). All rights reserved. Used with permission.)

COMPUTER-AIDED ANALYSIS
As we have just seen, PSpice can be enormously helpful in predicting the output of an op amp circuit, especially in the case of time-varying inputs. We will find, however, that our ideal op amp model agrees fairly well with PSpice simulations as a general rule.

When performing a PSpice simulation of an op amp circuit, we must be careful to remember that positive and negative dc supplies must be connected to the device (with the exception of the LM324, which is designed to be a single-supply op amp). Although the model shows the offset null pins used to zero the output voltage, PSpice does not build in any offset, so these pins are typically left floating (unconnected).

Table 6.3 shows the different op amp part numbers available in the Evaluation version of PSpice; other models are available in the commercial version of the software and from some manufacturers.

Simulate the circuit of Fig. 6.3 using PSpice. Determine the point(s) at which saturation begins if $\pm 15 \mathrm{~V}$ dc supplies are used to power the device. Compare the gain calculated by PSpice to what was predicted using the ideal op amp model.
We begin by drawing the inverting amplifier circuit of Fig. 6.3 using the schematic capture tool as shown in Fig. 6.32. Note that two separate 15 V dc supplies are required to power the op amp.


FIGURE 6.32 The inverting amplifier of Fig. 6.3 drawn using a $\mu \mathrm{A} 741$ op amp.
Our previous analysis using an ideal op amp model predicted a gain of -10 . With an input of $5 \sin 3 t \mathrm{mV}$, this led to an output voltage of $-50 \sin 3 t \mathrm{mV}$. However, an implicit assumption in the analysis was that any voltage input would be amplified by a factor of -10 . Based on practical considerations, we expect this to be true for small input voltages, but the output will eventually saturate to a value comparable to the corresponding power supply voltage.

We perform a dc sweep from -2 to +2 volts, as shown in Fig. 6.33; this is a slightly larger range than the supply voltage divided by the gain, so we expect our results to include the positive and negative saturation regions.

Using the cursor tool on the simulation results shown in Fig. 6.34a, the input-output characteristic of the amplifier is indeed linear over a wide input range, corresponding approximately to $-1.45<\mathrm{Vs}<+1.45 \mathrm{~V}$ (Fig. 6.34b): This range is slightly less than the range defined by dividing the positive and negative supply voltages by the gain. Outside this range, the output of the op amp saturates, with only a slight dependence on the input voltage. In the two saturation regions, then, the circuit does not perform as a linear amplifier.

Increasing the number of cursor digits (Tools, $\underline{\text { Options, }} \underline{\text { Number of }}$ Cursor Digits) to 10 , we find that at an input voltage of Vs $=1.0 \mathrm{~V}$, the


■ FIGURE 6.33 DC sweep setup window.

(a)

(b)

FIGURE 6.34 (a) Output voltage of the inverting amplifer circuit, with the onset of saturation identified with the cursor tool. (b) Close-up of the cursor window.
output voltage is -9.99548340 , slightly less than the value of -10 predicted from the ideal op amp model, and slightly different from the value of -9.999448 obtained in Example 6.6 using an analytical model. Still, the results predicted by the PSpice $\mu \mathrm{A} 741$ model are within a few
hundredths of a percent of either analytical model, demonstrating that the ideal op amp model is indeed a remarkably accurate approximation for modern operational amplifier integrated circuits.

## PRACTICE

6.7 Simulate the remaining op amp circuits described in this chapter, and compare the results to those predicted using the ideal op amp model.

### 6.6. COMPARATORS AND THE INSTRUMENTATION AMPLIFIER

## The Comparator

Every op amp circuit we have discussed up to now has featured an electrical connection between the output pin and the inverting input pin. This is known as closed-loop operation, and is used to provide negative feedback as discussed previously. Closed loop is the preferred method of using an op amp as an amplifier, as it serves to isolate the circuit performance from variations in the open-loop gain that arise from changes in temperature or manufacturing differences. There are a number of applications, however, where it is advantageous to use an op amp in an open-loop configuration. Devices intended for such applications are frequently referred to as comparators, as they are designed slightly differently from regular op amps in order to improve their speed in open-loop operation.

Figure $6.35 a$ shows a simple comparator circuit where a 2.5 V reference voltage is connected to the noninverting input, and the voltage being compared $\left(v_{\mathrm{in}}\right)$ is connected to the inverting input. Since the op amp has a very large open-loop gain $A$ ( $10^{5}$ or greater, typically, as seen in Table 6.3), it does not take a large voltage difference between the input terminals to drive it into saturation. In fact, a differential input voltage as small as the supply voltage divided by $A$ is required-approximately $\pm 120 \mu \mathrm{~V}$ in the case of the circuit in Fig. $6.35 a$ and $A=10^{5}$. The distinctive output of the comparator circuit is shown in Fig. 6.35b, where the response swings


■ FIGURE 6.35 (a) An example comparator circuit with a 2.5 V reference voltage. (b) Graph of input-output characteristic.
between positive and negative saturation, with essentially no linear "amplification" region. Thus, a positive 12 V output from the comparator indicates that the input voltage is less than the reference voltage, and a negative 12 V output indicates an input voltage greater than the reference. Opposite behavior is obtained if we connect the reference voltage to the inverting input instead.

## EXAMPLE 6.8



FIGURE 6.36 One possible design for the required circuit.

Design a circuit that provides a "logic 1 " 5 V output if a certain voltage signal drops below 3 V , and zero volts otherwise.

Since we want the output of our comparator to swing between 0 and 5 V , we will use an op amp with a single-ended +5 V supply, connected as shown in Fig. 6.36. We connect a +3 V reference voltage to the noninverting input, which may be provided by two 1.5 V batteries in series, or a suitable Zener diode reference circuit. The input voltage signal (designated $v_{\text {signal }}$ ) is then connected to the inverting input. In reality, the saturation voltage range of a comparator circuit will be slightly less than that of the supply voltages, so that some adjustment may be required in conjunction with simulation or testing.

## PRACTICE

6.8 Design a circuit that provides a 12 V output if a certain voltage ( $v_{\text {signal }}$ ) exceeds 0 V , and a -2 V output otherwise.

Ans: One possible solution is shown in Fig. 6.37.


FIGURE 6.37 One possible solution to
Practice Problem 6.8.

## The Instrumentation Amplifier

The basic comparator circuit acts on the voltage difference between the two input terminals to the device, although it does not technically amplify signals as the output is not proportional to the input. The difference amplifier of Fig. 6.10 also acts on the voltage difference between the inverting and noninverting inputs, and as long as care is taken to avoid saturation, does provide an output directly proportional to this difference. When dealing with a very small input voltage, however, a better alternative is a device
known as an instrumentation amplifier, which is actually three op amp devices in a single package.

An example of the common instrumentation amplifier configuration is shown in Fig. 6.38a, and its symbol is shown in Fig. 6.38b. Each input is fed directly into a voltage follower stage, and the output of both voltage followers is fed into a difference amplifier stage. It is particularly well suited to applications where the input voltage signal is very small (for example, on the order of millivolts), such as that produced by thermocouples or strain gauges, and where a significant common-mode noise signal of several volts may be present.


FIGURE 6.38 (a) The basic instrumentation amplifier. (b) Commonly used symbol.
If components of the instrumentation amplifier are fabricated all on the same silicon "chip," then it is possible to obtain well-matched device characteristics and to achieve precise ratios for the two sets of resistors. In order to maximize the CMRR of the instrumentation amplifier, we expect $R_{4} / R_{3}=R_{2} / R_{1}$, so that equal amplification of common-mode components of the input signals is obtained. To explore this further, we identify the voltage at the output of the top voltage follower as " $v_{-}$," and the voltage at the output of the bottom voltage follower as " $v_{+}$." Assuming all three op amps are ideal and naming the voltage at either input of the difference stage $v_{x}$, we may write the following nodal equations:

$$
\begin{equation*}
\frac{v_{x}-v_{-}}{R_{1}}+\frac{v_{x}-v_{\mathrm{out}}}{R_{2}}=0 \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{v_{x}-v_{+}}{R_{3}}+\frac{v_{x}}{R_{4}}=0 \tag{21}
\end{equation*}
$$

Solving Eq. [21] for $v_{x}$, we find that

$$
\begin{equation*}
v_{x}=\frac{v_{+}}{1+R_{3} / R_{4}} \tag{22}
\end{equation*}
$$

and upon substituting into Eq. [20], obtain an expression for $v_{\text {out }}$ in terms of the input:

$$
\begin{equation*}
v_{\mathrm{out}}=\frac{R_{4}}{R_{3}}\left(\frac{1+R_{2} / R_{1}}{1+R_{4} / R_{3}}\right) v_{+}-\frac{R_{2}}{R_{1}} v_{-} \tag{23}
\end{equation*}
$$

From Eq. [23] it is clear that the general case allows amplification of common-mode components to the two inputs. In the specific case where
$R_{4} / R_{3}=R_{2} / R_{1}=K$, however, Eq. [23] reduces to $K\left(v_{+}-v_{-}\right)=K v_{d}$, so that (asssuming ideal op amps) only the difference is amplified and the gain is set by the resistor ratio. Since these resistors are internal to the instrumentation amplifier and not accessible to the user, devices such as the AD622 allow the gain to be set anywhere in the range of 1 to 1000 by connecting an external resistor between two pins (shown as $R_{G}$ in Fig. 6.38b).

## SUMMARY AND REVIEW

In this chapter we introduced a new circuit element-a three-terminal device-called the operational amplifier (or more commonly, the op amp). In many circuit analysis situations it is approximated as an ideal device, which leads to two rules that are applied. We studied several op amp circuits in detail, including the inverting amplifier with gain $R_{f} / R_{1}$, the noninverting amplifer with gain $1+R_{f} / R_{1}$, and the summing amplifier. We were also introduced to the voltage follower and the difference amplifier, although the analysis of these two circuits was left for the reader. The concept of cascaded stages was found to be particularly useful, as it allows a design to be broken down into distinct units, each of which has a specific function. We took a slight detour and introduced briefly a two-terminal nonlinear circuit element, the Zener diode, as it provides a practical and straightforward voltage reference. We then used this element to contruct practical voltage and current sources using op amps, removing some of the mystery as to their origins.

Modern op amps have nearly ideal characteristics, as we found when we opted for a more detailed model based on a dependent source. Still, nonidealities are encountered occasionally, so we considered the role of negative feedback in reducing the effect of temperature and manufacturing-related variations in various parameters, common-mode rejection, and saturation. One of the most interesting nonideal characteristics of any op amp is slew rate. By simulating three different cases, we were able to see how the output voltage can struggle to follow the form of the input voltage signal once its frequency becomes high enough. We concluded the chapter with two special cases: the comparator, which intentionally makes use of our ability to saturate a practical (nonideal) op amp, and the instrumentation amplifier, which is routinely used to amplify very small voltages.

This is a good point to pause, take a breath, and recap some of the key points. At the same time, we will highlight relevant examples as an aid to the reader.

- There are two fundamental rules that must be applied when analyzing ideal op amp circuits:

1. No current ever flows into either input terminal. (Example 6.1)
2. No voltage ever exists between the input terminals.

- Op amp circuits are usually analyzed for an output voltage in terms of some input quantity or quantities. (Examples 6.1, 6.2)
- Nodal analysis is typically the best choice in analyzing op amp circuits, and it is usually better to begin at the input, and work toward the output. (Examples 6.1, 6.2)
- The output current of an op amp cannot be assumed; it must be found after the output voltage has been determined independently. (Example 6.2)
- The gain of an inverting op amp circuit is given by the equation

$$
v_{\mathrm{out}}=-\frac{R_{f}}{R_{1}} v_{\mathrm{in}}
$$

- The gain of a noninverting op amp circuit is given by the equation

$$
v_{\mathrm{out}}=\left(1+\frac{R_{f}}{R_{1}}\right) v_{\mathrm{in}}
$$

(Example 6.1)

- Cascaded stages may be analyzed one stage at a time to relate the output to the input. (Example 6.3)
- Zener diodes provide a convenient voltage reference. They are not symmetric, however, meaning the two terminals are not interchangeable. (Example 6.4)
- Op amps can be used to construct current sources which are independent of the load resistance over a specific current range. (Example 6.5)
- A resistor is almost always connected from the output pin of an op amp to its inverting input pin, which incorporates negative feedback into the circuit for increased stability.
- The ideal op amp model is based on the approximation of infinite open-loop gain $A$, infinite input resistance $R_{i}$, and zero output resistance $R_{o}$. (Example 6.6)
- In practice, the output voltage range of an op amp is limited by the supply voltages used to power the device. (Example 6.7)
- Comparators are op amps designed to be driven into saturation. These circuits operate in open loop, and hence have no external feedback resistor. (Example 6.8)


## READING FURTHER

Two very readable books which deal with a variety of op amp applications are:
R. Mancini (ed.), Op Amps Are For Everyone, 2nd ed. Amsterdam: Newnes, 2003. Also available on the Texas Instruments website (www.ti.com).
W. G. Jung, Op Amp Cookbook, 3rd ed. Upper Saddle River, N.J.: PrenticeHall, 1997.
Characteristics of Zener and other types of diodes are covered in Chapter 1 of
W. H. Hayt, Jr., and G. W. Neudeck, Electronic Circuit Analysis and Design, 2nd ed. New York: Wiley, 1995.
One of the first reports of the implementation of an "operational amplifier" can be found in
J. R. Ragazzini, R. M. Randall, and F. A. Russell, "Analysis of problems in dynamics by electronic circuits," Proceedings of the IRE 35(5), 1947, pp. 444-452.

And an early applications guide for the op amp can be found on the Analog Devices, Inc. website (www.analog.com):

George A. Philbrick Researches, Inc., Applications Manual for Computing Amplifiers for Modelling, Measuring, Manipulating \& Much Else.
Norwood, Mass.: Analog Devices, 1998.

## EXERCISES

### 6.2 The Ideal Op Amp

1. For the op amp circuit shown in Fig. 6.39, calculate $v_{\text {out }}$ if (a) $R_{1}=R_{2}=$ $100 \Omega$ and $v_{\text {in }}=5 \mathrm{~V}$; (b) $R_{2}=200 R_{1}$ and $v_{\text {in }}=1 \mathrm{~V}$; (c) $R_{1}=4.7 \mathrm{k} \Omega$, $R_{2}=47 \mathrm{k} \Omega$, and $v_{\mathrm{in}}=20 \sin 5 t \mathrm{~V}$.


FIGURE 6.39


FIGURE 6.40


FIGURE 6.41
2. Determine the power dissipated by a $100 \Omega$ resistor connected between ground and the output pin of the op amp of Fig. 6.39 if $v_{\text {in }}=4 \mathrm{~V}$ and (a) $R_{1}=2 R_{2}$;
(b) $R_{1}=1 \mathrm{k} \Omega$ and $R_{2}=22 \mathrm{k} \Omega$; (c) $R_{1}=100 \Omega$ and $R_{2}=101 \Omega$.
3. Connect a $1 \Omega$ resistor between ground and the output terminal of the op amp of Fig. 6.39, and sketch $v_{\text {out }}(t)$ if (a) $R_{1}=R_{2}=10 \Omega$ and $v_{\text {in }}=5 \sin 10 t \mathrm{~V}$; (b) $R_{1}=0.2 R_{2}=1 \mathrm{k} \Omega$, and $v_{\text {in }}=5 \cos 10 t \mathrm{~V}$; (c) $R_{1}=10 \Omega, R_{2}=200 \Omega$, and $v_{\text {in }}=1.5+5 e^{-t} \mathrm{~V}$.
4. For the circuit of Fig. 6.40, calculate $v_{\text {out }}$ if (a) $R_{1}=R_{2}=100 \mathrm{k} \Omega, R_{L}=100 \Omega$, and $v_{\text {in }}=5 \mathrm{~V}$; (b) $R_{1}=0.1 R_{2}, R_{L}=\infty$, and $v_{\text {in }}=2 \mathrm{~V}$; (c) $R_{1}=1 \mathrm{k} \Omega, R_{2}=0$, $R_{L}=1 \Omega$, and $v_{\text {in }}=43.5 \mathrm{~V}$.
5. (a) Design a circuit which converts a voltage $v_{1}(t)=9 \cos 5 t \mathrm{~V}$ into $9 \sin 5 t \mathrm{~V}$.
(b) Verify your design by analyzing the final circuit.
6. A certain load resistor requires a constant 5 V dc supply. Unfortunately, its resistance value changes with temperature. Design a circuit which supplies the requisite voltage if only 9 V batteries and standard $10 \%$ tolerance resistor values are available.
7. For the circuit of Fig. $6.40, R_{1}=R_{L}=50 \Omega$. Calculate the value of $R_{2}$ required to deliver 5 W to $R_{L}$ if $V_{\text {in }}$ equals (a) 5 V ; (b) 1.5 V . (c) Repeat parts $(a)$ and (b) if $R_{L}$ is reduced to $22 \Omega$.
8. Calculate $v_{\text {out }}$ as labeled in the schematic of Fig. 6.41 if (a) $i_{\text {in }}=1 \mathrm{~mA}, R_{p}=$ $2.2 \mathrm{k} \Omega$, and $R_{3}=1 \mathrm{k} \Omega$; (b) $i_{\text {in }}=2 \mathrm{~A}, R_{p}=1.1 \Omega$, and $R_{3}=8.5 \Omega$. (c) For each case, state whether the circuit is wired as a noninverting or an inverting amplifier. Explain your reasoning.
9. (a) Design a circuit using only a single op amp which adds two voltages $v_{1}$ and $v_{2}$ and provides an output voltage twice their sum (i.e., $v_{\text {out }}=2 v_{1}+2 v_{2}$ ).
(b) Verify your design by analyzing the final circuit.
10. (a) Design a circuit that provides a current $i$ which is equal in magnitude to the sum of three input voltages $v_{1}, v_{2}$, and $v_{3}$. (Compare volts to amperes.)
(b) Verify your design by analyzing the final circuit.
11. (a) Design a circuit that provides a voltage $v_{\text {out }}$ which is equal to the difference between two voltages $v_{2}$ and $v_{1}$ (i.e., $v_{\text {out }}=v_{2}-v_{1}$ ), if you have only the following resistors from which to choose: two $1.5 \mathrm{k} \Omega$ resistors, four $6 \mathrm{k} \Omega$ resistors, or three $500 \Omega$ resistors. (b) Verify your design by analyzing the final circuit.
12. Analyze the circuit of Fig. 6.42 and determine a value for $V_{1}$, which is referenced to ground.


- FIGURE 6.42

13. Derive an expression for $v_{\text {out }}$ as a function of $v_{1}$ and $v_{2}$ for the circuit represented in Fig. 6.43.


■ FIGURE 6.43
14. Explain what is wrong with each diagram in Fig. 6.44 if the two op amps are known to be perfectly ideal.

(a)

(b)

■ FIGURE 6.44
15. For the circuit depicted in Fig. 6.45, calculate $v_{\text {out }}$ if $I_{s}=2 \mathrm{~mA}, R_{Y}=4.7 \mathrm{k} \Omega$, $R_{X}=1 \mathrm{k} \Omega$, and $R_{f}=500 \Omega$.


FIGURE 6.45
16. Consider the amplifier circuit shown in Fig. 6.45. What value of $R_{f}$ will yield $v_{\text {out }}=2 \mathrm{~V}$ when $I_{s}=10 \mathrm{~mA}$ and $R_{Y}=2 R_{X}=500 \Omega$ ?
17. With respect to the circuit shown in Fig. 6.46, calculate $v_{\text {out }}$ if $v_{s}$ equals (a) $2 \cos 100 t \mathrm{mV}$; (b) $2 \sin \left(4 t+19^{\circ}\right) \mathrm{V}$.


FIGURE 6.46

### 6.3 Cascaded Stages

18. Calculate $v_{\text {out }}$ as labeled in the circuit of Fig. 6.47 if $R_{x}=1 \mathrm{k} \Omega$.

19. For the circuit of Fig. 6.47, determine the value of $R_{x}$ that will result in a value of $v_{\text {out }}=10 \mathrm{~V}$.
20. Referring to Fig. 6.48, sketch $v_{\text {out }}$ as a function of (a) $v_{\text {in }}$ over the range of $-2 \mathrm{~V} \leq v_{\text {in }} \leq+2 \mathrm{~V}$, if $R_{4}=2 \mathrm{k} \Omega$; (b) $R_{4}$ over the range of $1 \mathrm{k} \Omega \leq R_{4} \leq 10 \mathrm{k} \Omega$, if $v_{\text {in }}=300 \mathrm{mV}$.


FIGURE 6.48
21. Obtain an expression for $v_{\text {out }}$ as labeled in the circuit of Fig. 6.49 if $v_{1}$ equals (a) 0 V ; (b) 1 V ; (c) -5 V ; (d) $2 \sin 100 t \mathrm{~V}$.

22. The 1.5 V source of Fig. 6.49 is disconnected, and the output of the circuit shown in Fig. 6.48 is connected to the left-hand terminal of the $500 \Omega$ resistor instead. Calculate $v_{\text {out }}$ if $R_{4}=2 \mathrm{k} \Omega$ and (a) $v_{\text {in }}=2 \mathrm{~V}, v_{1}=1 \mathrm{~V}$; (b) $v_{\text {in }}=1 \mathrm{~V}$, $v_{1}=0 ;(c) v_{\text {in }}=1 \mathrm{~V}, v_{1}=-1 \mathrm{~V}$.
23. For the circuit shown in Fig. 6.50, compute $v_{\text {out }}$ if (a) $v_{1}=2 v_{2}=0.5 v_{3}=2.2 \mathrm{~V}$ and $R_{1}=R_{2}=R_{3}=50 \mathrm{k} \Omega$; (b) $v_{1}=0, v_{2}=-8 \mathrm{~V}, v_{3}=9 \mathrm{~V}$, and $R_{1}=0.5 R_{2}=$ $0.4 R_{3}=100 \mathrm{k} \Omega$.


FIGURE 6.50
24. (a) Design a circuit which will add the voltages produced by three separate pressure sensors, each in the range of $0 \leq v_{\text {sensor }} \leq 5 \mathrm{~V}$, and produce a positive voltage $v_{\text {out }}$ linearly correlated to the voltage sum such that $v_{\text {out }}=0$ when all three voltages are zero, and $v_{\text {out }}=2 \mathrm{~V}$ when all three voltages are at their maximum. (b) Verify your design by analyzing the final circuit.
25. (a) Design a circuit which produces an output voltage $v_{\text {out }}$ proportional to the difference of two positive voltages $v_{1}$ and $v_{2}$ such that $v_{\text {out }}=0$ when both voltages are equal, and $v_{\text {out }}=10 \mathrm{~V}$ when $v_{1}-v_{2}=1 \mathrm{~V}$. (b) Verify your design by analyzing the final circuit.
26. (a) Three pressure-sensitive sensors are used to double-check the weight readings obtained from the suspension systems of a long-range jet airplane. Each sensor is calibrated such that $10 \mu \mathrm{~V}$ corresponds to 1 kg . Design a circuit which adds the three voltage signals to produce an output voltage calibrated such that 10 V corresponds to $400,000 \mathrm{~kg}$, the maximum takeoff weight of the aircraft. (b) Verify your design by analyzing the final circuit.
27. (a) The oxygen supply to a particular bathysphere consists of four separate tanks, each equipped with a pressure sensor capable of measuring between 0 (corresponding to 0 V output) and 500 bar (corresponding to 5 V output). Design a circuit which produces a voltage proportional to the total pressure in all tanks, such that 1.5 V corresponds to 0 bar and 3 V corresponds to 2000 bar. (b) Verify your design by analyzing the final circuit.
28. For the circuit shown in Fig. 6.51, let $v_{\text {in }}=8 \mathrm{~V}$, and select values for $R_{1}, R_{2}$, and $R_{3}$ to ensure an output voltage $v_{\text {out }}=4 \mathrm{~V}$.

29. For the circuit of Fig. 6.52, derive an expression for $v_{\text {out }}$ in terms of $v_{\text {in }}$.


### 6.4 Circuits for Voltage and Current Sources

33. A particular passive network can be represented by a Thévenin equivalent resistance between $10 \Omega$ and $125 \Omega$ depending on the operating temperature. (a) Design a circuit which provides a constant 2.2 V to this network regardless of temperature. (b) Verify your design with an appropriate simulation (resistance can be varied from within a single simulation, as described in Chap. 8).
34. Calculate the voltage $V_{1}$ as labeled in the circuit of Fig. 6.53 if the battery is rated at $V_{\text {batt }}$ equal to (a) 9 V ; (b) 12 V . (c) Verify your solutions with appropriate simulations, commenting on the possible origin of any discrepancies.


■ FIGURE 6.53
35. (a) Design a current source based on the 1N750 diode which is capable of providing a dc current of $750 \mu \mathrm{~A}$ to a load $R_{L}$, such that $1 \mathrm{k} \Omega<R_{L}<50 \mathrm{k} \Omega$.
(b) Verify your design with an appropriate simulation (note that resistance can be varied within a single simulation, as described in Chap. 8).
36. (a) Design a current source able to provide a dc current of 50 mA to an unspecified load. Use a 1 N 4733 diode ( $V_{\mathrm{br}}=5.1 \mathrm{~V}$ at 76 mA ). (b) Use an appropriate simulation to determine the permissible range of load resistance for your design.
37. (a) Design a current source able to provide a dc current of 10 mA to an unspecified load. Use a 1 N 4747 diode ( $V_{\mathrm{br}}=20 \mathrm{~V}$ at 12.5 mA ). (b) Use an appropriate simulation to determine the permissible range of load resistance for your design.
38. The circuit depicted in Fig. 6.54 is known as a Howland current source. Derive expressions for $v_{\text {out }}$ and $I_{L}$, respectively as a function of $V_{1}$ and $V_{2}$.
39. For the circuit depicted in Fig. 6.54, known as a Howland current source, set $V_{2}=0, R_{1}=R_{3}$, and $R_{2}=R_{4}$; then solve for the current $I_{L}$ when $R_{1}=2 R_{2}=$ $1 \mathrm{k} \Omega$ and $R_{L}=100 \Omega$.

### 6.5 Practical Considerations

40. (a) Employ the parameters listed in Table 6.3 for the $\mu \mathrm{A} 741 \mathrm{op}$ amp to analyze the circuit of Fig. 6.55 and compute a value for $v_{\text {out. }}$ (b) Compare your result to what is predicted using the ideal op amp model.


■ FIGURE 6.55
41. (a) Employ the parameters listed in Table 6.3 for the $\mu \mathrm{A} 741 \mathrm{op}$ amp to analyze the circuit of Fig. 6.10 if $R=1.5 \mathrm{k} \Omega, v_{1}=2 \mathrm{~V}$, and $v_{2}=5 \mathrm{~V}$. (b) Compare your solution to what is predicted using the ideal op amp model.
42. Define the following terms, and explain when and how each can impact the performance of an op amp circuit: (a) common-mode rejection ratio; (b) slew rate; (c) saturation; (d) feedback.
43. For the circuit of Fig. 6.56, replace the $470 \Omega$ resistor with a short circuit, and compute $v_{\text {out }}$ using (a) the ideal op amp model; $(b)$ the parameters listed in Table 6.3 for the $\mu \mathrm{A} 741 \mathrm{op} \mathrm{amp}$; (c) an appropriate PSpice simulation.
(d) Compare the values obtained in parts $(a)$ to $(c)$ and comment on the possible origin of any discrepancies.
44. If the circuit of Fig. 6.55 is analyzed using the detailed model of an op amp (as opposed to the ideal op amp model), calculate the value of open-loop gain $A$ required to achieve a closed-loop gain within $2 \%$ of its ideal value.
45. Replace the 2 V source in Fig. 6.56 with a sinusoidal voltage source having a magnitude of 3 V and radian frequency $\omega=2 \pi f$. (a) Which device, a $\mu \mathrm{A} 741$ op amp or an LF411 op amp, will track the source frequency better over the range $1 \mathrm{~Hz}<f<10 \mathrm{MHz}$ ? Explain. (b) Compare the frequency performance of the circuit over the range $1 \mathrm{~Hz}<f<10 \mathrm{MHz}$ using appropriate PSpice simulations, and compare the results to your prediction in part $(a)$.
46. (a) For the circuit of Fig. 6.56, if the op amp (assume LF411) is powered by matched 9 V supplies, estimate the maximum value to which the $470 \Omega$ resistor can be increased before saturation effects become apparent. (b) Verify your prediction with an appropriate simulation.
47. For the circuit of Fig. 6.55, calculate the differential input voltage and the input bias current if the op amp is $\mathrm{a}(\mathrm{n})(a) \mu \mathrm{A} 741$; (b) LF411; (c) AD549K; (d) OPA690.
48. Calculate the common-mode gain for each device listed in Table 6.3. Express your answer in units of $V / V$, not $d B$.


FIGURE 6.54


■ FIGURE 6.56


■ FIGURE 6.57


FIGURE 6.58

### 6.6 Comparators and the Instrumentation Amplifier

49. Human skin, especially when damp, is a reasonable conductor of electricity. If we assume a resistance of less than $10 \mathrm{M} \Omega$ for a fingertip pressed across two terminals, design a circuit which provides a +1 V output if this nonmechanical switch is "closed" and -1 V if it is "open."
50 . Design a circuit which provides an output voltage $v_{\text {out }}$ based on the behavior of another voltage $v_{\text {in }}$, such that
$v_{\text {out }}=\left\{\begin{aligned}+2.5 \mathrm{~V} & v_{\text {in }}>1 \mathrm{~V} \\ 1.2 \mathrm{~V} & \text { otherwise }\end{aligned}\right.$
50. For the instrumentation amplifier shown in Fig. 6.38a, assume that the three internal op amps are ideal, and determine the CMRR of the circuit if
(a) $R_{1}=R_{3}$ and $R_{2}=R_{4}$; (b) all four resistors have different values.
51. For the circuit depicted in Fig. 6.57, sketch the expected output voltage $v_{\text {out }}$ as a function of $v_{\text {active }}$ for $-5 \mathrm{~V} \leq v_{\text {active }} \leq+5 \mathrm{~V}$, if $v_{\text {ref }}$ is equal to $(a)-3 \mathrm{~V}$; (b) +3 V .
52. For the circuit depicted in Fig. 6.58, (a) sketch the expected output voltage $v_{\text {out }}$ as a function of $v_{1}$ for $-5 \mathrm{~V} \leq v_{1} \leq+5 \mathrm{~V}$, if $v_{2}=+2 \mathrm{~V}$; $(b)$ sketch the expected output voltage $v_{\text {out }}$ as a function of $v_{2}$ for $-5 \mathrm{~V} \leq v_{2} \leq+5 \mathrm{~V}$, if $v_{1}=+2 \mathrm{~V}$.
53. For the circuit depicted in Fig. 6.59, sketch the expected output voltage $v_{\text {out }}$ as a function of $v_{\text {active }}$, if $-2 \mathrm{~V} \leq v_{\text {active }} \leq+2 \mathrm{~V}$. Verify your solution using a $\mu \mathrm{A} 741$ (although it is slow compared to op amps designed specifically for use as comparators, its PSpice model works well, and as this is a dc application speed is not an issue). Submit a properly labeled schematic with your results.


FIGURE 6.59
55. In digital logic applications, a +5 V signal represents a logic " 1 " state, and a 0 V signal represents a logic " 0 " state. In order to process real-world information using a digital computer, some type of interface is required, which typically includes an analog-to-digital (A/D) converter-a device that converts analog signals into digital signals. Design a circuit that acts as a simple 1-bit A/D, with any signal less than 1.5 V resulting in a logic " 0 " and any signal greater than 1.5 V resulting in a logic " 1. .
56. A common application for instrumentation amplifiers is to measure voltages in resistive strain gauge circuits. These strain sensors work by exploiting the changes in resistance that result from geometric distortions, as in Eq. [6] of Chap. 2. They are often part of a bridge circuit, as shown in Fig. $6.60 a$, where the strain gauge is identified as $R_{G}$. (a) Show that $V_{\text {out }}=V_{\text {in }}\left[\frac{R_{2}}{R_{1}+R_{2}}-\frac{R_{3}}{R_{3}+R_{\text {Gauge }}}\right]$. (b) Verify that $V_{\text {out }}=0$ when the three fixed-value resistors $R_{1}, R_{2}$, and $R_{3}$ are all chosen to be equal to the unstrained gauge resistance $R_{\text {Gauge. }}$. (c) For the intended application, the gauge selected has an unstrained resistance of $5 \mathrm{k} \Omega$, and a maximum resistance increase of
$50 \mathrm{~m} \Omega$ is expected. Only $\pm 12 \mathrm{~V}$ supplies are available. Using the instrumentation amplifier of Fig. 6.60 b , design a circuit that will provide a voltage signal of +1 V when the strain gauge is at its maximum loading.

(a)

FIGURE 6.60

## AD622 Specifications

Amplifier gain $G$ can be varied from 2 to 1000 by connecting a resistor between pins 1 and 8 with a value calculated by $R=\frac{50.5}{G-1} \mathrm{k} \Omega$.

(b)
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## Chapter-Integrating Exercises

57. (a) You're given an electronic switch which requires 5 V at 1 mA in order to close; it is open with no voltage present at its input. If the only microphone available produces a peak voltage of 250 mV , design a circuit which will energize the switch when someone speaks into the microphone. Note that the audio level of a general voice may not correspond to the peak voltage of the microphone. (b) Discuss any issues that may need to be addressed if your circuit were to be implemented.
58. You've formed a band, despite advice to the contrary. Actually, the band is pretty good except for the fact that the lead singer (who owns the drum set, the microphones, and the garage where you practice) is a bit tone-deaf. Design a circuit that takes the output from each of the five microphones your band uses, and adds the voltages to create a single voltage signal which is fed to the amplifer. Except not all voltages should be equally amplified. One microphone output should be attenuated such that its peak voltage is $10 \%$ of any other microphone's peak voltage.
59. Cadmium sulfide (CdS) is commonly used to fabricate resistors whose value depends on the intensity of light shining on the surface. In Fig. 6.61 a CdS "photocell" is used as the feedback resistor $R_{f}$. In total darkness, it has a resistance of $100 \mathrm{k} \Omega$, and a resistance of $10 \mathrm{k} \Omega$ under a light intensity of 6 candela. $R_{L}$ represents a circuit that is activated when a voltage of 1.5 V or less is applied to its terminals. Choose $R_{1}$ and $V_{s}$ so that the circuit represented by $R_{L}$ is activated by a light of 2 candela or brighter.


■ FIGURE 6.61
60. A fountain outside a certain office building is designed to reach a maximum height of 5 meters at a flow rate of $100 \mathrm{l} / \mathrm{s}$. A variable position valve in line with the water supply to the fountain can be controlled electrically, such that 0 V applied results in the valve being fully open, and 5 V results in the valve being closed. In adverse wind conditions the maximum height of the fountain needs to be adjusted; if the wind velocity exceeds $50 \mathrm{~km} / \mathrm{h}$, the height cannot exceed 2 meters. A wind velocity sensor is available which provides a voltage calibrated such that 1 V corresponds to a wind velocity of $25 \mathrm{~km} / \mathrm{h}$. Design a circuit which uses the velocity sensor to control the fountain according to specifications.
61. For the circuit of Fig. 6.43 , let all resistor values equal $5 \mathrm{k} \Omega$. Sketch $v_{\text {out }}$ as a function of time if (a) $v_{1}=5 \sin 5 t \mathrm{~V}$ and $v_{2}=5 \cos 5 t \mathrm{~V}$; (b) $v_{1}=4 e^{-t} \mathrm{~V}$ and $v_{2}=5 e^{-2 t} \mathrm{~V}$; (c) $v_{1}=2 \mathrm{~V}$ and $v_{2}=e^{-t} \mathrm{~V}$.


[^0]:    Ans: $v_{\text {out }}=v_{2}-v_{1}$. Hint: Use voltage division to obtain $v_{b}$.

[^1]:    (2) Pounds per square inch, absolute. This is a differential pressure measurement relative to a vacuum reference.

