2 Circuit Analysis Techniques

2.1 Thévenin and Norton Equivalent Circuits

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Problems

In Chapter 1 the basic electric circuit concepts were presented. In this chapter we consider some circuit analysis techniques, since one needs not only basic knowledge but also practical and efficient techniques for solving problems associated with circuit operations.

One simplifying technique often used in complex circuit problems is that of breaking the circuit into pieces of manageable size and analyzing individually the pieces that may be already familiar. *Equivalent* circuits are introduced which utilize Thévenin's and Norton's theorems to replace a voltage source by a current source or vice versa. *Nodal* and *loop* analysis methods are then presented. Later the principles of *superposition* and *linearity* are discussed. Also, *wye–delta* transformation is put forth as a tool for network reduction. Finally, computer-aided circuit analyses with SPICE and MATLAB are introduced. The chapter ends with a case study of practical application.

2.1 THÉVENIN AND NORTON EQUIVALENT CIRCUITS

For a *linear* portion of a circuit consisting of ideal sources and linear resistors, the volt–ampere (v–i) relationship at any two accessible terminals can be expressed by the linear equation

$$v = Ai + B \tag{2.1.1}$$

where A and B are two constants. The Thévenin equivalent circuit at any two terminals a and b (to replace the linear portion of the circuit) is given by

$$v = R_{\mathrm{Th}} i + v_{\mathrm{oc}} \tag{2.1.2}$$

where it can be seen that

$$R_{\mathrm{Th}} = v/i|_{v_{\mathrm{oc}}=0} \tag{2.1.3}$$

and

$$v_{\mathrm{oc}} = v|_{i=0} \tag{2.1.4}$$

Thus, v_{oc} is known as the *open-circuit voltage* (or Thévenin voltage) with $i = 0$, and R_{Th} is the *Thévenin equivalent resistance* (as seen from the terminals a–b) with $v_{\mathrm{oc}} = 0$. Equation (2.1.4) accounts for the ideal sources present in that linear portion of the circuit, as shown in Figure 2.1.1(a), whereas Equation (2.1.3) implies deactivating or zeroing all ideal sources (i.e., replacing voltage sources by short circuits and current sources by open circuits). The model with the voltage source v_{oc} in series with R_{Th} is known as *Thévenin equivalent circuit*, as shown in Figure 2.1.1(b).

Equation (2.1.1) may be rewritten as

$$i = \frac{v}{A} - \frac{B}{A} = \frac{v}{R_{\mathrm{Th}}} - \frac{v_{\mathrm{oc}}}{R_{\mathrm{Th}}} = \frac{v}{R_{\mathrm{Th}}} - i_{\mathrm{sc}} \tag{2.1.5}$$

which is represented by the *Norton equivalent circuit* with a current source i_{sc} in parallel with R_{Th}, as shown in Figure 2.1.1(c). Notice that with $v = 0$, $i = -i_{\mathrm{sc}}$. Also, $i_{\mathrm{sc}} = v_{\mathrm{oc}}/R_{\mathrm{Th}}$, or $v_{\mathrm{oc}} = i_{\mathrm{sc}} R_{\mathrm{Th}}$.

Besides representing complete one-ports (or two-terminal networks), Thévenin and Norton equivalents can be applied to portions of a network (with respect to any two terminals) to simplify intermediate calculations. Moreover, successive conversions back and forth between the two equivalents often save considerable labor in circuit analysis with multiple sources. *Source transformations* can be used effectively by replacing the voltage source V with a series resistance R by an equivalent current source $I (= V/R)$ in parallel with the same resistance R, or vice versa.

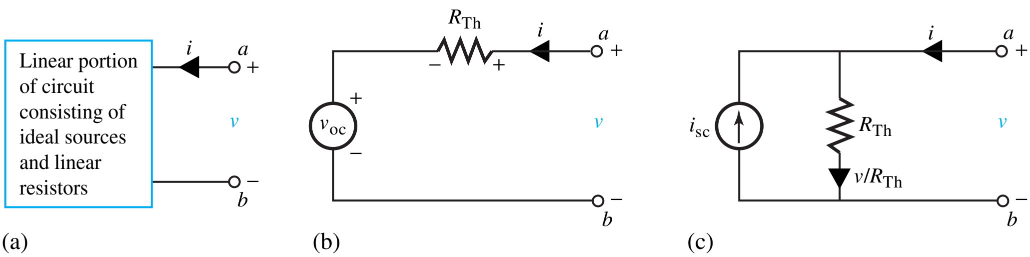


Figure 2.1.1 Equivalent circuits. **(a)** Two-terminal or one-port network. **(b)** Thévenin equivalent circuit. **(c)** Norton equivalent circuit.

EXAMPLE 2.1.1

Consider the circuit shown in Figure E2.1.1(a). Reduce the portion of the circuit to the left of terminals a–b to (a) a Thévenin equivalent and (b) a Norton equivalent. Find the current through $R = 16\ \Omega$, and comment on whether resistance matching is accomplished for maximum power transfer.

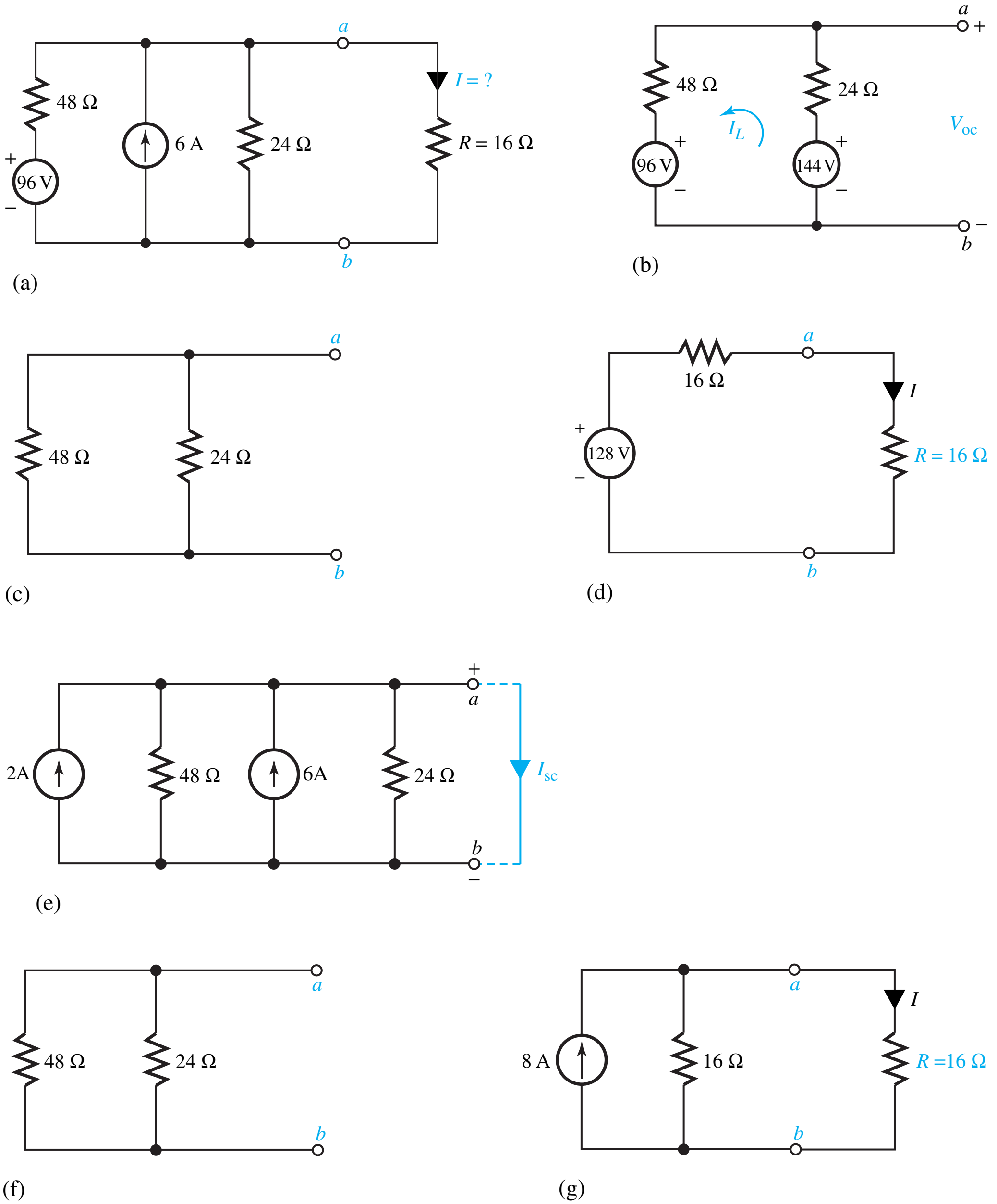


Figure E2.1.1

Solution

The 6-A source with 24 Ω in parallel can be replaced by a voltage source of $6 \times 24 = 144$ V with 24 Ω in series. Thus, by using source transformation, in terms of voltage sources, the equivalent circuit to the left of terminals *a*–*b* is shown in Figure E2.1.1(b).

(a) KVL: $144 - 24I_L - 48I_L - 96 = 0$, or $72I_L = 48$, or $I_L = {}^2\!/_3$ A

$$V_{oc} = 144 - 24(^2\!/_3) = 128 \text{ V}$$

Deactivating or zeroing all ideal sources, i.e., replacing voltage sources by short circuits in the present case, the circuit of Figure E2.1.1(b) reduces to that shown in Figure E2.1.1(c).

Viewed from terminals *a–b*, the 48-Ω resistor and the 24-Ω resistor are in parallel,

$$R_{Th} = 48 \| 24 = \frac{48 \times 24}{48 + 24} = 16 \ \Omega$$

Thus, the Thévenin equivalent to the left of terminals *a–b*, attached with the 16-Ω resistor, is shown in Figure E2.1.1(d). Note that the Thévenin equivalent of any linear circuit consists of a single Thévenin voltage source in series with a single equivalent Thévenin resistance.

The current in the 16-Ω resistor to the right of terminals *a–b* can now be found,

$$I = 128/32 = 4 \text{ A}$$

(b) The 96-V source with 48 Ω in series can be replaced by a current source of $96/48 = 2$ A with a parallel resistance of 48 Ω. Thus, by using source transformation, in terms of current sources, the equivalent circuit to the left of terminals *a–b* is given in Figure E2.1.1(e).

Shorting terminals *a–b*, one can find I_{sc}, $I_{sc} = 8$ A. Replacing current sources by open circuits, viewed from terminals *a–b*, $R_{Th} = 48 \| 24 = 16\ \Omega$, which is the same as in part (a). The circuit of Figure E2.1.1(e) to the left of terminals *a–b* reduces to that shown in Figure E2.1.1(f).

Thus, the Norton equivalent to the left of terminals *a–b,* attached with the 16-Ω resistor, is given in Figure E2.1.1(g). Note that the Norton equivalent of any linear circuit consists of a single current source in parallel with a single equivalent Thévenin resistance.

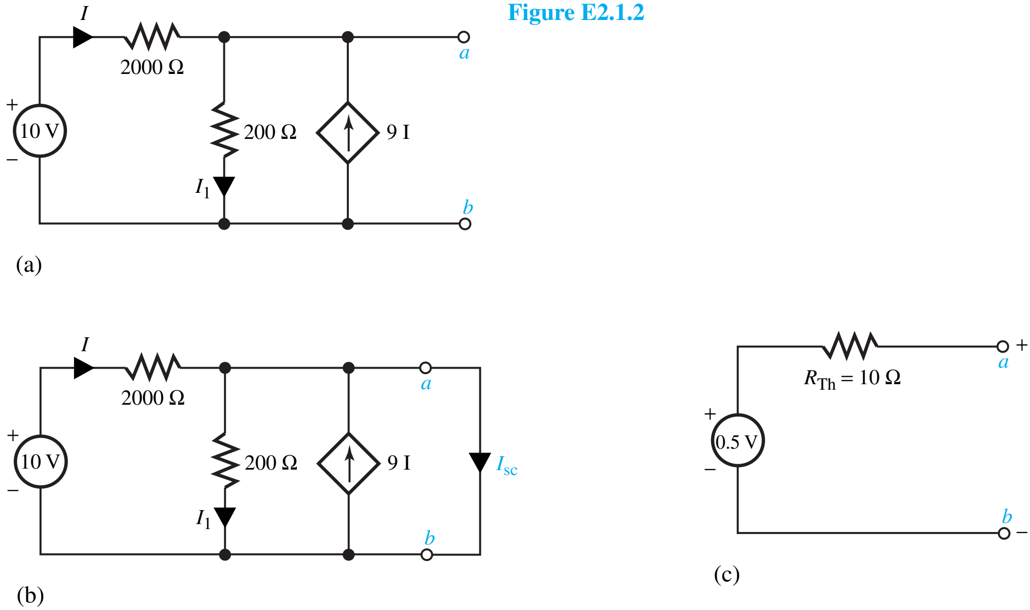
The current in the 16-Ω resistor to the right of terminals *a–b* can now be found. $I = 4$ A, which is the same as in part (a).

The equivalent source resistance, also known as the output resistance, is the same as the load resistance of 16 Ω in the present case. Hence, resistance matching is accomplished for maximum power transfer.

EXAMPLE 2.1.2

Consider the circuit of Figure E2.1.2(a), including a dependent source. Obtain the Thévenin equivalent at terminals *a–b*.

Figure E2.1.2



Solution

First, the open-circuit voltage at terminals a–b is to be found.

$$\text{KCL at node } a: \quad I + 9I = I_1, \ \text{ or } I_1 = 10I$$

$$\text{KVL for the left-hand mesh:} \quad 2000I + 200I_1 = 10, \ \text{ or } 4000I = 10, \ \text{ or } I = 1/400 \text{ A}$$

$$V_{oc} = 200I_1 = 200(1/400) = 0.5 \text{ V}$$

Because of the presence of a dependent source, in order to find R_{Th}, one needs to determine I_{sc} after shorting terminals a–b, as shown in Figure E2.1.2(b).

Note that $I_1 = 0$, since $V_{ab} = 0$.

$$\text{KCL at node } a: \quad I_{sc} = 9I + I = 10I$$

$$\text{KVL for the outer loop:} \quad 2000I = 10, \ \text{ or } I = 1/200 \text{ A}$$

$$I_{sc} = 10(1/200) = 1/20 \text{ A}$$

Hence the equivalent Thévenin resistance R_{Th} viewed from terminals a–b is

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{0.5}{1/20} = 10\Omega$$

Thus, the Thévenin equivalent is given in Figure E2.1.2(c).

The preceding examples illustrate how a complex network could be reduced to a simple representation at an output port. The effect of load on the terminal behavior or the effect of an output load on the network can easily be evaluated. Thévenin and Norton equivalent circuits help us in matching, for example, the speakers to the amplifier output in a stereo system. Such equivalent circuit concepts permit us to represent the entire system (generation and distribution) connected to a receptacle (plug or outlet) in a much simpler model with the open-circuit voltage as the measured voltage at the receptacle itself.

When a system of sources is so large that its voltage and frequency remain constant regardless of the power delivered or absorbed, it is known as an *infinite bus*. Such a bus (node) has a voltage and a frequency that are unaffected by external disturbances. The infinite bus is treated as an ideal voltage source. Even though, for simplicity, only resistive networks are considered in this section, the concept of equivalent circuits is also employed in ac sinusoidal steady-state circuit analysis of networks consisting of inductors and capacitors, as we shall see in Chapter 3.

2.2 NODE-VOLTAGE AND MESH-CURRENT ANALYSES

The node-voltage and mesh-current methods, which complement each other, are well-ordered systematic methods of analysis for solving complicated network problems. The former is based on the KCL equations, whereas the KVL equations form the basis for the latter. In both methods an appropriate number of simultaneous algebraic equations are developed. The unknown nodal voltages are found in the nodal method, whereas the unknown mesh currents are calculated in the loop (or mesh) method. A decision to use one or the other method of analysis is usually based on the number of equations needed for each method.

Even though, for simplicity, only resistive networks with dc voltages are considered in this section, the methods themselves are applicable to more general cases with time-varying sources, inductors, capacitors, and other circuit elements.

Nodal-Voltage Method

A set of node-voltage variables that implicitly satisfy the KVL equations is selected in order to formulate circuit equations in this nodal method of analysis. A *reference* (datum) node is chosen arbitrarily based on convenience, and from each of the remaining nodes to the reference node, the voltage drops are defined as *node-voltage* variables. The circuit is then described completely by the necessary number of KCL equations whose solution yields the unknown nodal voltages from which the voltage and the current in every circuit element can be determined. Thus, the number of simultaneous equations to be solved will be equal to one less than the number of network nodes. All voltage sources in series with resistances are replaced by equivalent current sources with conductances in parallel. In general, resistances may be replaced by their corresponding conductances for convenience. Note that the nodal-voltage method is a general method of network analysis that can be applied to any network.

Let us illustrate the method by considering the simple, but typical, example shown in Figure 2.2.1. By replacing the voltage sources with series resistances by their equivalent current sources with shunt conductances, Figure 2.2.1 is redrawn as Figure 2.2.2, in which one can identify three nodes, *A, B,* and *O*.

Notice that the voltages V_{AO}, V_{BO}, and V_{AB} satisfy the KVL relation:

$$V_{AB} + V_{BO} - V_{AO} = 0, \qquad \text{or} \qquad V_{AB} = V_{AO} - V_{BO} = V_A - V_B \tag{2.2.1}$$

where the node voltages V_A and V_B are the voltage drops from A to O and B to O, respectively. With node O as reference, and with V_A and V_B as the node-voltage unknown variables, one can write the two independent KCL equations:

Node *A:* $$V_A G_1 + (V_A - V_B)G_3 = I_1, \quad \text{or} \quad (G_1 + G_3)V_A - G_3 V_B = I_1 \tag{2.2.2}$$

Node *B:* $$V_B G_2 - (V_A - V_B)G_3 = I_2, \quad \text{or} \quad -G_3 V_A + (G_2 + G_3)V_B = I_2 \tag{2.2.3}$$

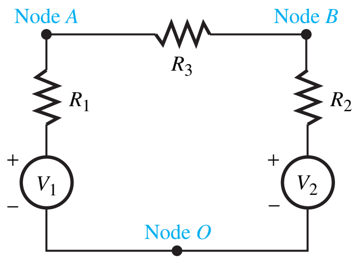


Figure 2.2.1 Circuit for illustration of nodal-voltage method.

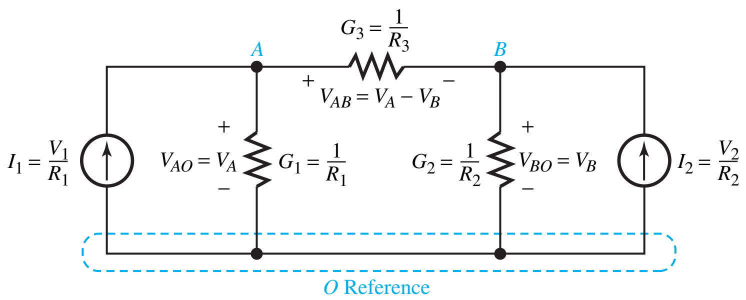


Figure 2.2.2 Redrawn Figure 2.2.1 for node-voltage method of analysis.

An examination of these equations reveals a pattern that will allow nodal equations to be written directly by inspection by following the rules given here for a network containing no dependent sources.

1. For the equation of node A, the coefficient of V_A is the positive sum of the conductances connected to node A; the coefficient of V_B is the negative sum of the conductances connected between nodes A and B. The right-hand side of the equation is the sum of the current sources feeding into node A.
2. For the equation of node B, a similar situation exists. Notice the coefficient of V_B to be the positive sum of the conductances connected to node B; the coefficient of V_A is the negative sum of the conductances connected between B and A. The right-hand side of the equation is the sum of the current sources feeding into node B.

Such a formal systematic procedure will result in a set of N independent equations of the following form for a network with $(N + 1)$ nodes containing no dependent sources:

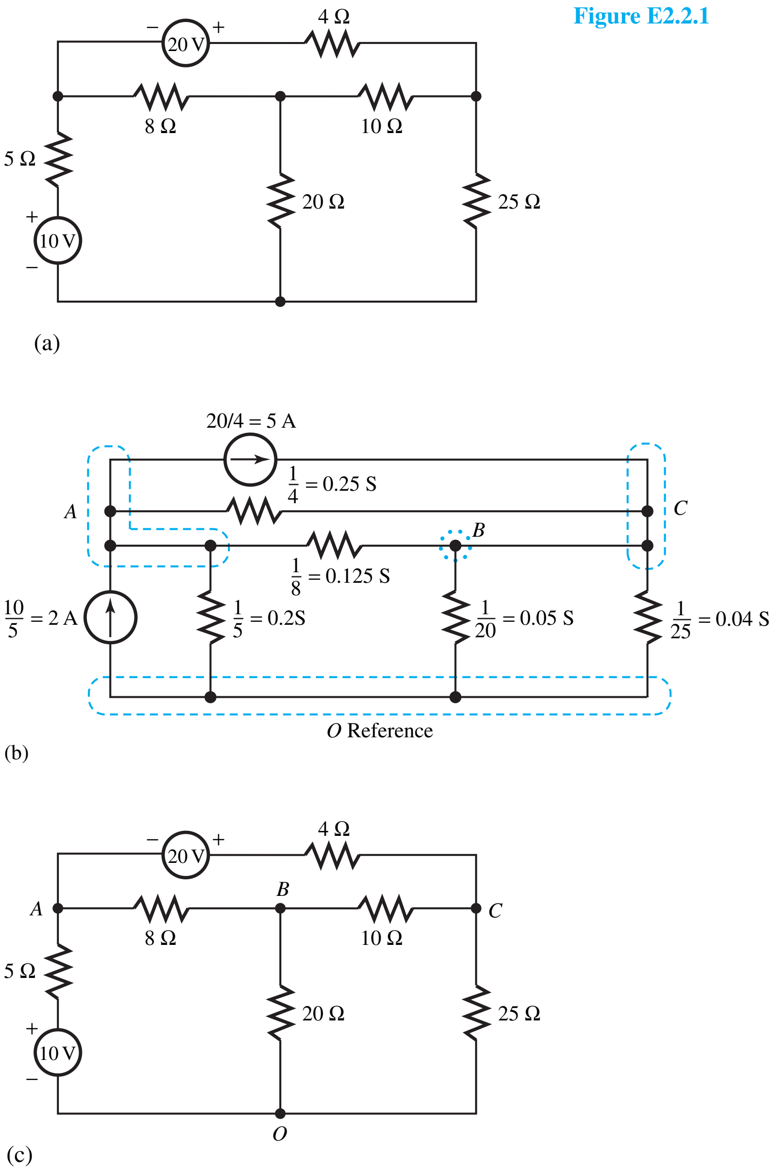
$$\begin{array}{rcrcccccl}
G_{11}V_1 & - & G_{12}V_2 & - & \cdots & - & G_{1N}V_N & = & I_1 \\
-G_{21}V_1 & + & G_{22}V_2 & - & \cdots & - & G_{2N}V_N & = & I_2 \\
\vdots & & & & & & & \vdots & \\
-G_{N1}V_1 & - & G_{N2}V_2 & - & \cdots & + & G_{NN}V_N & = & I_N
\end{array} \tag{2.2.4}$$

where G_{NN} is the sum of all conductances connected to node N, $G_{JK} = G_{KJ}$ is the sum of all conductances connected between nodes J and K, and I_N is the sum of all current sources entering node N. By solving the equations for the unknown node voltages, other voltages and currents in the circuit can easily be determined.

EXAMPLE 2.2.1

By means of nodal analysis, find the current delivered by the 10-V source and the voltage across the 10-Ω resistance in the circuit shown in Figure E2.2.1(a).

Figure E2.2.1



Solution

STEP 1: Replace all voltage sources with series resistances by their corresponding Norton equivalents consisting of current sources with shunt conductances. The given circuit is redrawn in Figure E2.2.1(b) by replacing all resistors by their equivalent conductances.

STEP 2: Identify the nodes and choose a convenient reference node O. This is also shown in Figure E2.2.1(b).

STEP 3: In terms of unknown node-voltage variables, write the KCL equations at all nodes (except, of course, the reference node) by following rules 1 and 2 for nodal equations given in this section.

$$\begin{aligned}
&\text{Node } A: && (0.2+0.125+0.25)V_A - 0.125V_B - 0.25V_C &&= 2-5=-3\\
&\text{Node } B: && -0.125V_A + (0.125+0.05+0.1)V_B - 0.1V_C &&= 0\\
&\text{Node } C: && -0.25V_A - 0.1V_B + (0.25+0.1+0.04)V_C &&= 5
\end{aligned}$$

Rearranging, one gets

$$\begin{array}{rcrcrcr}
0.575\ V_A & - & 0.125\ V_B & - & 0.25\ V_C & = & -3\\
-0.125\ V_A & + & 0.275\ V_B & - & 0.1\ V_C & = & 0\\
-0.25\ V_A & - & 0.1\ V_B & + & 0.39\ V_C & = & 5
\end{array}$$

STEP 4: Simultaneously solve the independent equations for the unknown nodal voltages by Gauss elimination or Cramer's rule. In our example, the solution yields

$$V_A = 4.34\ V; \qquad V_B = 8.43V; \qquad V_C = 17.77\ \text{V}$$

STEP 5: Obtain the desired voltages and currents by the application of KVL and Ohm's law. To find the current I in the 10-V source, since it does not appear in Figure E2.2.1(b) redrawn for nodal analysis, one has to go back to the original circuit and identify the equivalence between nodes A and O, as shown in Figure E2.2.1(c).

Now one can solve for I, delivered by the 10-V source,

$$V_A = 4.34 = -5I + 10 \qquad \text{or} \qquad I = \frac{5.66}{5} = 1.132\ \text{A}$$

The voltage across the 10-Ω resistance is $V_B - V_C = 8.43 - 17.77 = -9.34$ V. The negative sign indicates that node C is at a higher potential than node B with respect to the reference node O.

Nodal analysis deals routinely with current sources. When we have voltage sources along with series resistances, the source-transformation technique may be used effectively to convert the voltage source to a current source, as seen in Example 2.2.1. However, in cases where we have *constrained nodes*, that is, the difference in potential between the two node voltages is constrained by a voltage source, the concept of a *supernode* becomes useful for the circuit analysis, as shown in the following illustrative example.

EXAMPLE 2.2.2

For the network shown in Figure E2.2.2, find the current in each resistor by means of nodal analysis.

Solution

Note that the reference node is chosen at one end of an independent voltage source, so that the node voltage V_A is known at the start,

$$V_A = 12\ \text{V}$$

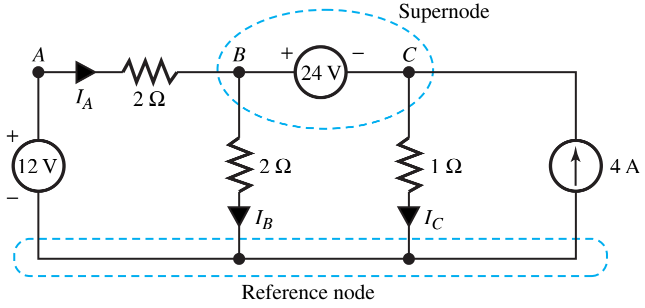


Figure E2.2.2

Note that we cannot express the branch current in the voltage source as a function of V_B and V_C. Here we have constrained nodes B and C. Nodal voltages V_B and V_C are not independent. They are related by the constrained equation

$$V_B - V_C = 24 \text{ V}$$

Let us now form a *supernode,* which includes the voltage source and the two nodes B and C, as shown in Figure E2.2.2. KCL must hold for this supernode, that is, the algebraic sum of the currents entering or leaving the supernode must be zero. Thus one valid equation for the network is given by

$$I_A - I_B - I_C + 4 = 0 \qquad \text{or} \qquad \frac{12 - V_B}{2} - \frac{V_B}{2} - \frac{V_C}{1} + 4 = 0$$

which reduces to

$$V_B + V_C = 10$$

This equation together with the supernode constraint equation yields

$$V_B = 17 \text{ V} \qquad \text{and} \qquad V_C = -7 \text{ V}$$

The currents in the resistors are thus given by

$$I_A = \frac{12 - V_B}{2} = \frac{12 - 17}{2} = -2.5 \text{ A}$$

$$I_B = \frac{V_B}{2} = \frac{17}{2} = 8.5 \text{ A}$$

$$I_C = \frac{V_C}{1} = \frac{-7}{1} = -7 \text{ A}$$

Mesh-Current Method

This complements the nodal-voltage method of circuit analysis. A set of independent *mesh-current* variables that implicitly satisfy the KCL equations is selected in order to formulate circuit equations in this mesh analysis. An *elementary loop*, or a *mesh*, is easily identified as one of the "window panes" of the whole circuit. However, it must be noted that not all circuits can be laid out to contain only meshes as in the case of planar networks. Those which cannot are called nonplanar circuits, for which the mesh analysis cannot be applied, but the nodal analysis can be employed.

A mesh current is a fictitious current, which is defined as the one circulating around a mesh of the circuit in a certain direction. While the direction is quite arbitrary, a clockwise direction is traditionally chosen. Branch currents can be found in terms of mesh currents, whose solution is obtained from the independent simultaneous equations. The number of necessary equations in the mesh-analysis method is equal to the number of independent loops or meshes.

All current sources with shunt conductances will be replaced by their corresponding Thévenin equivalents consisting of voltage sources with series resistances. Let us illustrate the method by considering a simple, but typical, example, as shown in Figure 2.2.3.

Replacing the current source with shunt resistance by the Thévenin equivalent, Figure 2.2.3 is redrawn as Figure 2.2.4, in which one can identify two elementary loops, or independent meshes.

By assigning loop or mesh-current variables I_1 and I_2, as shown in Figure 2.2.4, both in the clockwise direction, one can write the KVL equations for the two closed paths (loops) *ABDA* and *BCDB*,

$$\text{Loop } ABDA\text{:}\quad I_1R_1 + (I_1 - I_2)R_2 = V_1 - V_2 \quad \text{or} \quad (R_1 + R_2)I_1 - R_2I_2 = V_1 - V_2 \qquad (2.2.5)$$

$$\text{Loop } BCDB\text{:}\quad I_2R_3 + (I_2 - I_1)R_2 = V_2 - V_3 \quad \text{or} \quad -R_2I_1 + (R_2 + R_3)I_2 = V_2 - V_3 \qquad (2.2.6)$$

Notice that current I_1 exists in R_1 and R_2 in the direction indicated; I_2 exists in R_2 and R_3 in the direction indicated; hence, the net current in R_2 is $I_1 - I_2$ directed from B to D. An examination of Equations (2.2.5) and (2.2.6) reveals a pattern that will allow loop equations to be written directly by inspection by following these rules:

1. In the first loop equation with mesh current I_1, the coefficient of I_1 is the sum of the resistances in that mesh; the coefficient of I_2 is the negative sum of the resistances common to both meshes. The right-hand side of the equation is the algebraic sum of the source voltage rises taken in the direction of I_1.
2. Similar statements can be made for the second loop with mesh current I_2. (See also the similarity in setting up the equations for the mesh-current and nodal-voltage methods of analysis.)

Such a formal systematic procedure will yield a set of N independent equations of the following form for a network with N independent meshes containing no dependent sources:

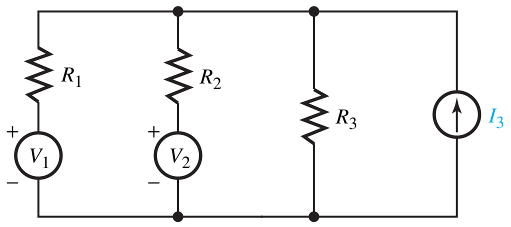


Figure 2.2.3 Circuit for illustration of mesh-current method.

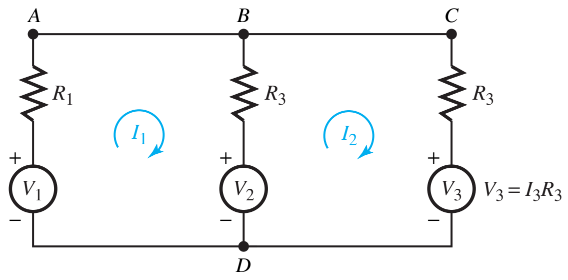


Figure 2.2.4 Redrawn Figure 2.2.3 for mesh-current method of analysis.

$$
\begin{array}{rcrcccrcl}
R_{11}I_1 & - & R_{12}I_2 & - & \cdots & - & R_{1N}I_N & = & V_1 \\
-R_{21}I_1 & + & R_{22}V_2 & - & \cdots & - & R_{2N}I_N & = & V_2 \\
\vdots & & & & & & & \vdots & \\
-R_{N1}I_1 & - & R_{N2}V_2 & - & \cdots & + & R_{NN}I_N & = & V_N
\end{array}
\tag{2.2.7}
$$

where R_{NN} is the sum of all resistances contained in mesh N, $R_{JK} = R_{KJ}$ is the sum of all resistances common to both meshes J and K, and V_N is the algebraic sum of the source-voltage rises in mesh N, taken in the direction of I_N.

By solving the equations for the unknown mesh currents, other currents and voltages in the circuit elements can be determined easily.

EXAMPLE 2.2.3

By means of mesh-current analysis, obtain the current in the 10-V source and the voltage across the 10-Ω resistor in the circuit of Example 2.2.1.

Solution

STEP 1: Replace all current sources with shunt resistances by their corresponding Thévenin equivalents consisting of voltage sources with series resistances. Conductances included in the circuit are replaced by their equivalent resistances.

In this example, since there are no current sources and conductances, the circuit of Figure E2.2.1(a) is redrawn as Figure E2.2.3 for convenience.

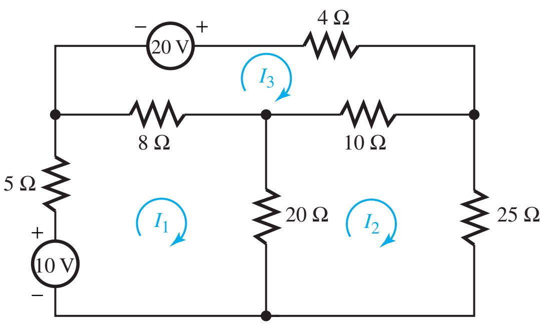


Figure E2.2.3

STEP 2: Identify elementary loops (meshes) and choose a mesh-current variable for each elementary loop, with all loop currents in the same clockwise direction. Mesh currents I_1, I_2, and I_3 are shown in Figure E2.2.3.

STEP 3: In terms of unknown mesh-current variables, write the KVL equations for all meshes by following the rules for mesh analysis.

$$
\begin{array}{lr}
\text{Loop 1 with mesh current } I_1: & (5+8+20)I_1 - 20I_2 - 8I_3 = 10 \\
\text{Loop 2 with mesh current } I_2: & -20I_1 + (20+10+25)I_2 - 10I_3 = \;\;0 \\
\text{Loop 3 with mesh current } I_3: & -8I_1 - 10I_2 + (4+10+8)I_3 = 20
\end{array}
$$

Rearranging, one gets

$$\begin{aligned} 33I_1 - 20I_2 - 8I_3 &= 10 \\ -20I_1 + 55I_2 - 10I_3 &= 0 \\ -8I_1 - 10I_2 + 22I_3 &= 20 \end{aligned}$$

STEP 4: Simultaneously solve the independent equations for the unknown mesh currents by Gauss elimination or Cramer's rule.

In this example the solution yields

$$I_1 = 1.132 \text{ A}; \qquad I_2 = 0.711 \text{ A}; \qquad I_3 = 1.645 \text{ A}$$

The current through the 10-V source is $I_1 = 1.132$ A, which is the same as in Example 2.2.1. The voltage across the 10-Ω resistor is $V_{BC} = 10(I_2 - I_3) = 10(0.711 - 1.645) = -9.34$ V, which is the same as in Example 2.2.1.

Looking at Examples 2.2.1 and 2.2.3, it can be seen that there is no specific advantage for either method since the number of equations needed for the solution is three in either case. Such may not be the case in a number of other problems, in which case one should choose judiciously the more convenient method, usually with the lower number of equations to be solved.

The mesh-current method deals routinely with voltage sources. When we have current sources with shunt conductances, the source-transformation technique may be used effectively to convert the current source to a voltage source. However, in cases where we have *constrained meshes*, that is, the two mesh currents are constrained by a current source, the concept of a *supermesh* becomes useful for the circuit analysis, as shown in the following illustrative example.

EXAMPLE 2.2.4

For the network shown in Figure E2.2.4, find the current delivered by the 10-V source and the voltage across the 3-Ω resistor by means of mesh-current analysis.

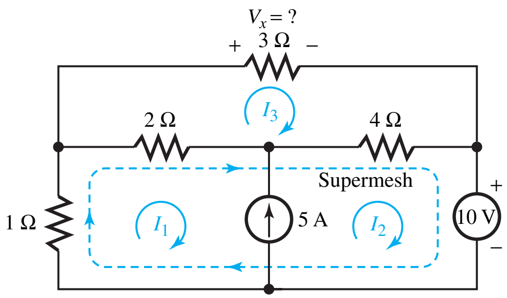


Figure E2.2.4

Solution

Note that we cannot express the voltage across the current source in terms of the mesh currents I_1 and I_2. The current source does, however, *constrain* the mesh currents by the following equation:

$$I_2 - I_1 = 5$$

Let us now form a *supermesh,* which includes meshes 1 and 2, as shown in Figure E2.2.4. We now write a KVL equation around the periphery of meshes 1 and 2 combined. This yields

$$1I_1 + 2(I_1 - I_3) + 4(I_2 - I_3) + 4(I_2 - I_3) + 10 = 0$$

Next we write a KVL equation for mesh 3,

$$3I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

Now we have the three linearly independent equations needed to find the three mesh currents I_1, I_2, and I_3. The solution of the three simultaneous equations yields

$$I_1 = \frac{-25A}{9}\ \text{A}; \qquad I_2 = \frac{20}{9}\ \text{A}; \qquad I_3 = \frac{70}{27}\ \text{A}$$

The current delivered by the 10-V source is $-I_2$, or $-20/9$ A. That is to say, the 10-V source is absorbing the current 20/9 A.

The voltage across the 3-Ω resistor is $V_x = 3I_3 = 3(70/27) = 70/9 = 7.78$ V.

Node-Voltage and Mesh-Current Equations with Controlled Sources

Since a controlled source acts at its terminals in the same manner as does an independent source, source conversion and application of KCL and KVL relations are treated identically for both types of sources. Because the strength of a controlled source depends on the value of a voltage or current elsewhere in the network, a *constraint* equation is written for each controlled source. After combining the constraint equations with the loop or nodal equations based on treating all sources as independent sources, the resultant set of equations are solved for the unknown current or voltage variables.

EXAMPLE 2.2.5

Consider the circuit in Figure E2.2.5(a), which include a controlled source, and find the current in the 5-V source and the voltage across the 5-Ω resistor by using (a) the loop-current method and (b) the node-voltage method.

Solution

(a) Loop-Current Method: The voltage-controlled current source and its parallel resistance are converted into a voltage-controlled voltage source and series resistance. When you are source transforming dependent sources, note that the identity of the control variable (i.e., the location in the circuit) must be retained. The converted circuit is shown in Figure E2.2.5(b) with the chosen loop currents I_1 and I_2.

The KVL equations are

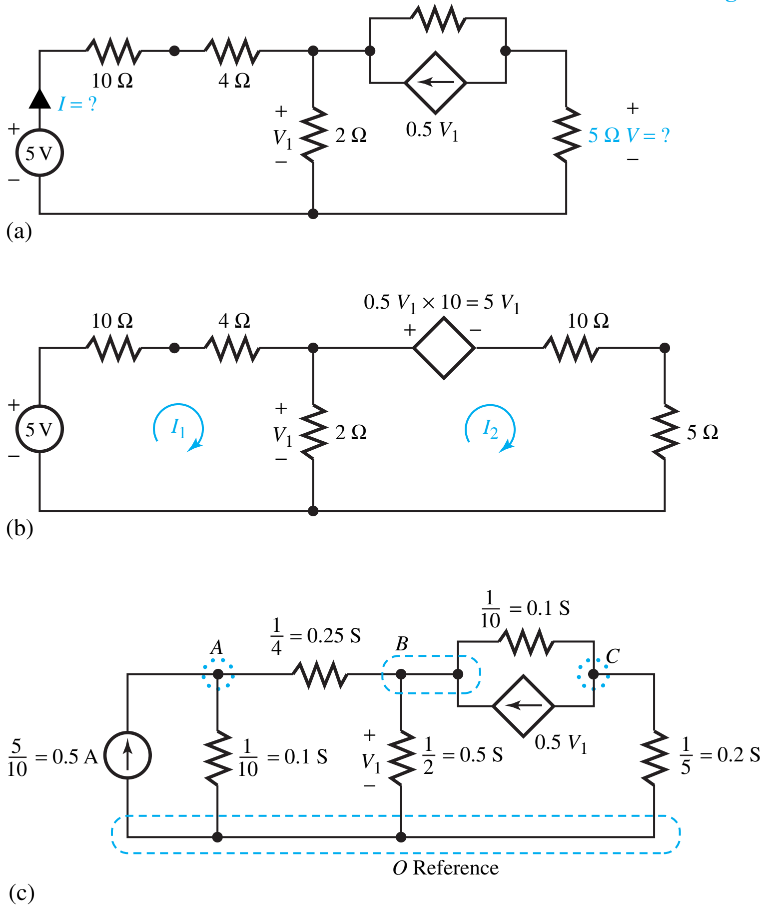
For loop carrying I_1: $\quad (10 + 4 + 2)I_1 - 2I_2 = 5$

For loop carrying I_2: $\quad -2I_1 + (2 + 10 + 5)I_2 = -5V_1$

The constraint equation is

$$V_1 = (I_1 - I_2)2$$

Figure E2.2.5



Combining the constraint equation with the loop equations, one gets

$$16I_1 - 2I_2 = 5; \qquad -2I_1 + 17I_2 = -10(I_1 - I_2), \qquad \text{or} \qquad 8I_1 + 7I_2 = 0$$

from which

$$I_1 = 35/128\ A; \qquad I_2 = -5/16\ A$$

Thus, the current through the 5-V source is $I = I_1 = 35/128 = 0.273$ A, and the voltage across the 5-Ω resistor is $V = 5I_2 = 5(-5/16) = -1.563$ V.

(b) Node-Voltage Method: The 5-V voltage source with its 10-Ω series resistor is replaced by its Norton equivalent. Resistances are converted into conductances and the circuit is redrawn in Figure E2.2.5(c) with the nodes shown.

The nodal equations are

$$A: \qquad (0.1 + 0.25)V_A - 0.25V_B = 0.5$$

$$B: \qquad -0.25V_A + (0.25 + 0.5 + 0.1)V_B - 0.1V_C = 0.5V_1$$

$$C: \qquad -0.1V_B + (0.1 + 0.2)V_C = -0.5V_1$$

The constraint equation is

$$V_1 = V_B$$

Combining these with the nodal equations already written, one has

$$\begin{array}{rcrcrcl} 0.35\ V_A & - & 0.25\ V_B & & & = & 0.5 \\ -0.25\ V_A & + & 0.35\ V_B & - & 0.1\ V_C & = & 0 \\ & & 0.4\ V_B & + & 0.3\ V_C & = & 0 \end{array}$$

Solving, one gets

$$V_A = 2.266V; \qquad V_B = 1.173; \qquad V_C = -1.564\ \text{V}$$

Notice that $V_C = -1.564$ V is the voltage V across the 5-Ω resistor, which is almost the same as that found in part (a).

In order to find the current I through the 5-V source, one needs to go back to the original circuit and recognize that

$$5 - 10I = V_A = 2.266 \qquad \text{or} \qquad I = 0.273\ \text{A}$$

which is the same as that found in part (a).

2.3 SUPERPOSITION AND LINEARITY

Mathematically a function is said to be linear if it satisfies two properties: *homogeneity* (*proportionality* or *scaling*) and *additivity* (*superposition*),

$$f\,(Kx) = Kf\,(x) \qquad \text{(homogeneity)} \tag{2.3.1}$$

where K is a scalar constant, and

$$f\,(x_1 + x_2) = f\,(x_1) + f\,(x_2) \qquad \text{(additivity)} \tag{2.3.2}$$

Linearity requires both additivity and homogeneity. For a linear circuit or system in which excitations x_1 and x_2 produce responses y_1 and y_2, respectively, the application of K_1x_1 and K_2x_2 together (i.e., $K_1x_1 + K_2x_2$) results in a response of $(K_1y_1 + K_2y_2)$, where K_1 and K_2 are constants. With the cause-and-effect relation between the excitation and the response, all *linear* systems satisfy the principle of *superposition*. A circuit consisting of independent sources, linear dependent sources, and linear elements is said to be a linear circuit. Note that a resistive element is linear. Capacitors and inductors are also circuit elements that have a linear input–output relationship provided that their initial stored energy is zero. Nonzero initial conditions are to be treated as independent sources.

In electric circuits, the excitations are provided by the voltage and current sources, whereas the responses are in terms of element voltages and currents. All circuits containing only ideal resistances, capacitances, inductances, and sources are linear circuits (described by linear differential equations). For a linear network consisting of several *independent* sources, according to the principle of superposition, the net response in any element is the algebraic sum of the individual responses produced by each of the independent sources acting only by itself. While each independent source acting on the network is considered separately by itself, the other independent sources are suppressed; that is to say, voltage sources are replaced by short circuits and current sources are replaced by open circuits, thereby reducing the source strength to zero. The effect of any dependent sources, however, must be included in evaluating the response due to each of the independent sources, as illustrated in the following example.

EXAMPLE 2.3.1

Determine the voltage across the 20-Ω resistor in the following circuit of Figure E2.3.1 (a) with the application of superposition.

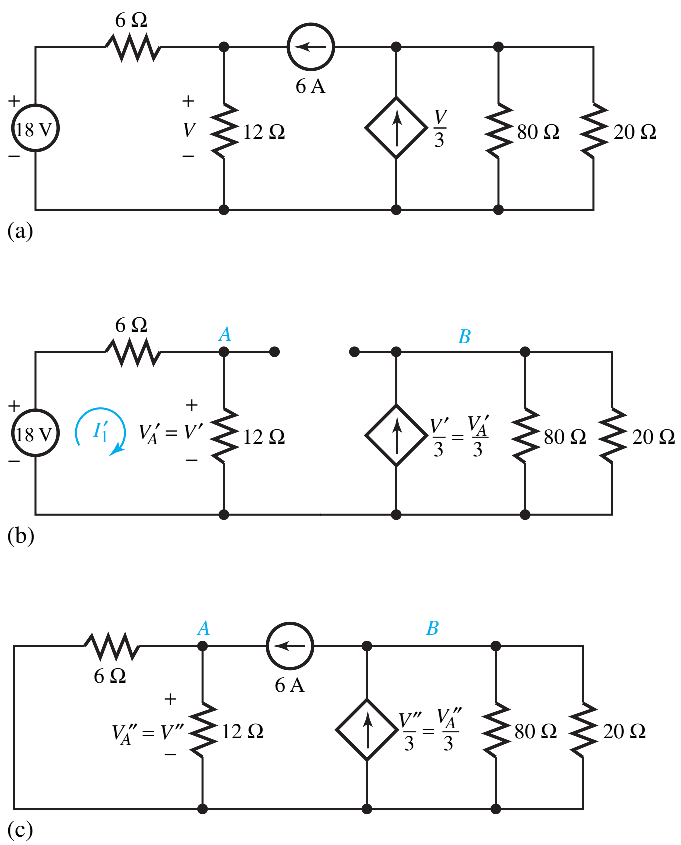


Figure E2.3.1

Solution

Let us suppress the independent sources in turn, recognizing that there are two independent sources. First, by replacing the independent current source with an open circuit, the circuit is drawn in Figure E2.3.1(b). Notice the designation of V' across the 12-Ω resistor and $V'/3$ as the dependent current source for this case. At node B,

$$\left(\frac{1}{80}+\frac{1}{20}\right) V'_B = \frac{V'_A}{3} \qquad \text{or} \qquad V'_B = \frac{V'_A}{48}$$

For the mesh on the left-hand side, $(6+12)I'_1 = 18$, or $I'_1 = 1$ A. But, $I'_1 = V'_A/12$, or $V'_A = 12$ V.

The voltage across the 20-Ω resistor from this part of the solution is

$$V'_B = \frac{12}{48} = \frac{1}{4}\ \text{V}$$

Next, by replacing the independent voltage source with a short circuit, the circuit is shown in Figure E2.3.1(c). Notice the designation of V'' across the 12-Ω resistor and $V''/3$ as the dependent current source for this case. At node A,

$$\left(\frac{1}{6}+\frac{1}{12}\right) V_A'' = 6 \qquad \text{or} \qquad V_A'' = 24 \text{ V}$$

and at node B,

$$\left(\frac{1}{80}+\frac{1}{20}\right) V_B'' = \frac{V_A''}{3} - 6 = \frac{24}{3} - 6 = 2 \qquad \text{or} \qquad V_B'' = 32 \text{ V}$$

Thus, the voltage across the 20-Ω resistor for this part of the solution is

$$V_B'' = 32 \text{ V}$$

Then the total net response, by superposition, is

$$V_B = V_B' + V_B'' = \frac{1}{4} + 32 = 32.25 \text{ V}$$

The *principle of superposition* is indeed a powerful tool for analyzing a wide range of linear systems in electrical, mechanical, civil, or industrial engineering.

2.4 WYE-DELTA TRANSFORMATION

Certain network configurations cannot be reduced or simplified by series–parallel combinations alone. In some such cases wye–delta (Y–Δ) transformation can be used to replace three resistors in wye configuration by three resistors in delta configuration, or vice versa, so that the networks are equivalent in so far as the terminals (A, B, C) are concerned, as shown in Figure 2.4.1.

For equivalence, it can be shown that (see Problem 2.4.1)

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}; \qquad R_B = \frac{R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}};$$
$$R_C = \frac{R_{CA}R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \tag{2.4.1}$$

$$R_{AB} = \frac{R_AR_B+R_BR_C+R_CR_A}{R_C}; \qquad R_{BC} = \frac{R_AR_B+R_BR_C+R_CR_A}{R_A};$$
$$R_{CA} = \frac{R_AR_B+R_BR_C+R_CR_A}{R_B} \tag{2.4.2}$$

For the simple case when $R_A = R_B = R_C = R_Y$, and $R_{AB} = R_{BC} = R_{CA} = R_\Delta$, Equations (2.4.1) and (2.4.2) become

$$R_Y = \frac{R_\Delta}{3} \tag{2.4.3}$$

$$R_\Delta = 3R_Y \tag{2.4.4}$$

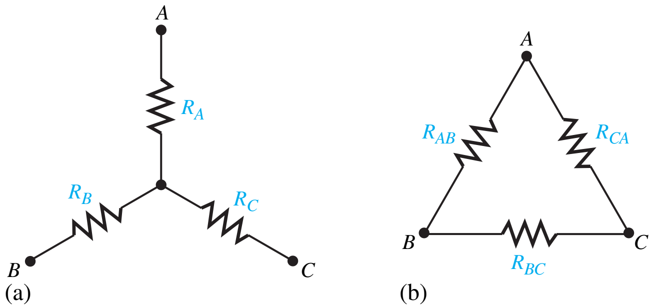


Figure 2.4.1 Wye–delta transformation. **(a)** Wye configuration. **(b)** Delta configuration.

EXAMPLE 2.4.1

Use delta–wye transformation for network reduction and determine the current through the 12-Ω resistor in the circuit of Figure E2.4.1(a).

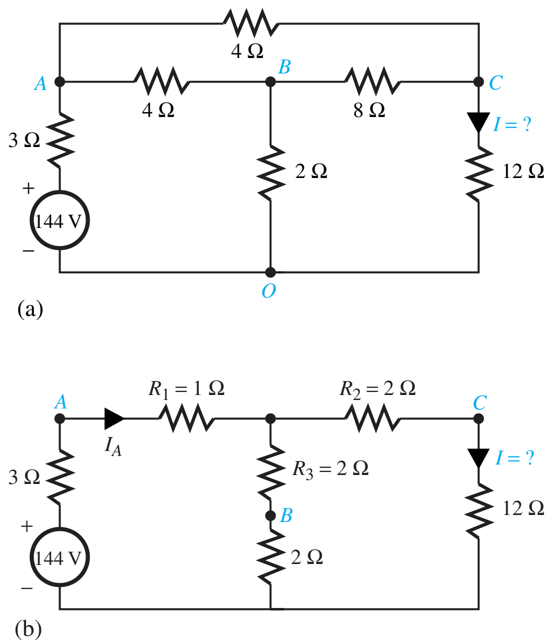


Figure E2.4.1

Solution

The delta-connected portion between terminals A–B–C is replaced by an equivalent wye connection [see Equation (2.4.1)] with

$$R_1 = \frac{4 \times 4}{4 + 8 + 4} = 1\ \Omega$$

$$R_2 = \frac{4 \times 8}{4 + 8 + 4} = 2\ \Omega$$

$$R_3 = \frac{4 \times 8}{4 + 8 + 4} = 2\ \Omega$$

The circuit is redrawn in Figure E2.4.1(b).

Using the KVL equation,

$$I_A = \frac{144}{(3 + 1) + (4 \parallel 14)} = \frac{81}{4}\ \text{A}$$

By current division,

$$I = \frac{81}{4} \times \frac{4}{18} = \frac{9}{2} = 4.5\ \text{A}$$

2.5 COMPUTER-AIDED CIRCUIT ANALYSIS: SPICE

A word of caution is appropriate if this is the student's first experience with simulation. Just as the proliferation of calculators did not eliminate the need to understand the theory of mathematics, circuit simulation programs do not eliminate the need to understand circuit theory. However, computer-aided tools can free the engineer from tedious calculations, thereby freeing more time for doing the kind of creative work a computer cannot do.

A circuit-analysis program known as SPICE, an acronym for simulation program with integrated circuit emphasis, is introduced in this section. The original SPICE program was developed in the early 1970s at the University of California at Berkeley. Since that time, various SPICE-based commercial products have been developed for personal computer and workstation platforms.[1]

A block diagram summarizing the major features of a SPICE-based circuit simulation program is shown in Figure 2.5.1. Micro Sim Corporation has developed a design center in

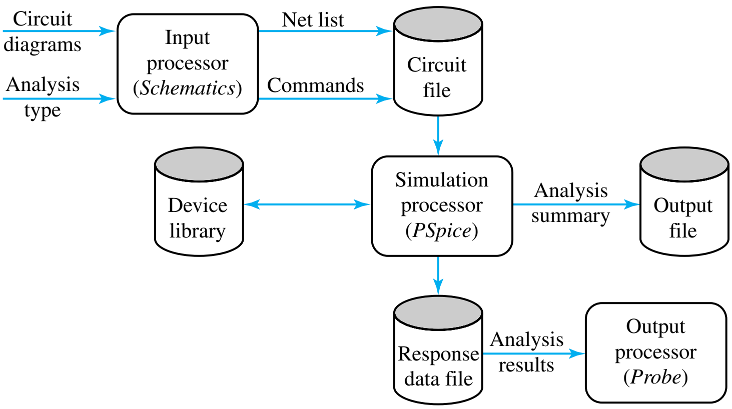


Figure 2.5.1 Major features of a SPice-based circuit simulation program.

[1]For supplementary reading on SPICE, the student is encouraged to refer to G. Roberts and A. Sedra, *SPICE*, 2nd ed., published by Oxford University Press (1997), and to P. Tuinenga, *SPICE*, 3rd ed., published by Prentice Hall (1995).

which the input processor is called *Schematics*, the simulation processor is a version of SPICE called *PSpice*, and the output processor is called *PROBE*. These three programs, working together, create a graphical environment in which the circuit diagram and the analysis objectives are entered using *Schematics*, the circuit is analyzed using *PSpice*, and the resulting circuit responses are viewed using *PROBE*. A student's version of these programs is widely available and is used in this book.

The first step for describing the circuit is to number the circuit nodes. The reference node (or ground node) is labeled as zero (0), and in PSpice syntax the other node names can be numbers or letters. In order to describe the circuit, statements are written with a separate statement for each circuit element. The name of an element must begin with a particular letter identifying the kind of circuit element. Some of these are listed here:

R	Resistor
V	Independent voltage source
I	Independent current source
G	Voltage-controlled current source
E	Voltage-controlled voltage source
F	Current-controlled current source
H	Current-controlled voltage source

While the original SPICE recognized only uppercase letters, PSpice is actually case insensitive. Because PSpice does not recognize subscripts, R_1, for example, will be represented by R1, and so on. The name of each circuit element must be unique. Numerical values can be specified in the following forms:

4567 or 4567.0 or 4.567 E3

SPICE uses the following scale factor designations:

T = 1E12	G = 1E9	MEG = 1E6
K = 1E3	M = 1E − 3	U = IE − 6
N = 1E − 9	P = 1E − 12	F = 1E − 15

Sometimes, for clarity, additional letters following a numerical value may be used; but these are ignored by SPICE. For example, 4.4 KOHMS is recognized as the value 4400, and “ohms” is ignored by the program. Comment statements are identified by an asterisk (*) in the first column, and these are helpful for making the program meaningful to users. PSpice also allows inserting comments on any line by starting the comment with a semicolon. Figure 2.5.2 shows the four types of controlled sources and their corresponding PSpice statements.

While SPICE is capable of several types of analysis, here we illustrate how to solve resistive circuits containing dc sources using the DC command. PSpice can *sweep* the value of the source, when the starting value, the end value, and the increment between values are given. If the starting and end values are the same, the solution is carried out for only a single value of the source.

Next we give an example of PSpice analysis. Note that SPICE has capabilities far beyond what we use in this section, and clearly, one can easily solve complex networks by using programs like PSpice.

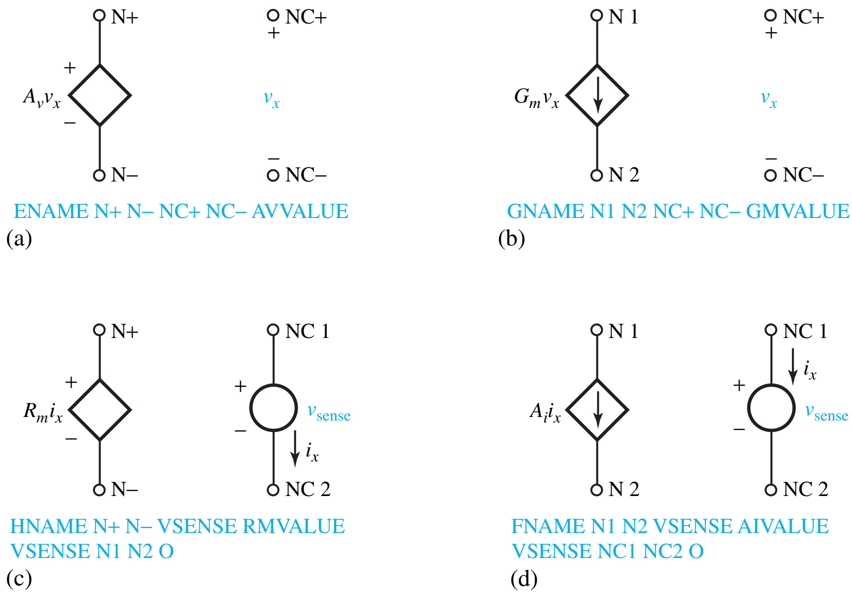


Figure 2.5.2 Four types of controlled sources and their corresponding PSpice statements. **(a)** Voltage-controlled voltage source. **(b)** Voltage-controlled current source. **(c)** Current-controlled voltage source. **(d)** Current-controlled current source.

EXAMPLE 2.5.1

Develop and execute a PSpice program to solve for the current I_2 in Figure E2.5.1(a).

Solution

Figure E2.5.1(b) is drawn showing the node numbers, and adding a voltage source of zero value in series with R_1, because there is a current-controlled source. The program is as follows:

```
EXAMPLE E2.5.1(a) A Title Identifying the Program.
* THE CIRCUIT DIAGRAM IS GIVEN IN FIGURE E2.5.1(b); a comment
statement
* CIRCUIT DESCRIPTION WITH COMPONENT STATEMENTS
IS 0 1 3
R1 1 4 5
R2 1 2 10
R3 2 0 2
R4 3 0 5
HCCVS 2 3 VSENSE 2
VSENSE 4 3 0
* ANALYSIS REQUEST
• DC IS 3 3 1
* OUTPUT REQUEST
• PRINT DC I(R2) V(1) V(2) V(3)
• END ; an end statement
```

After executing the program, from the output file, $I_2 = I(\mathrm{R2}) = 0.692$ A.

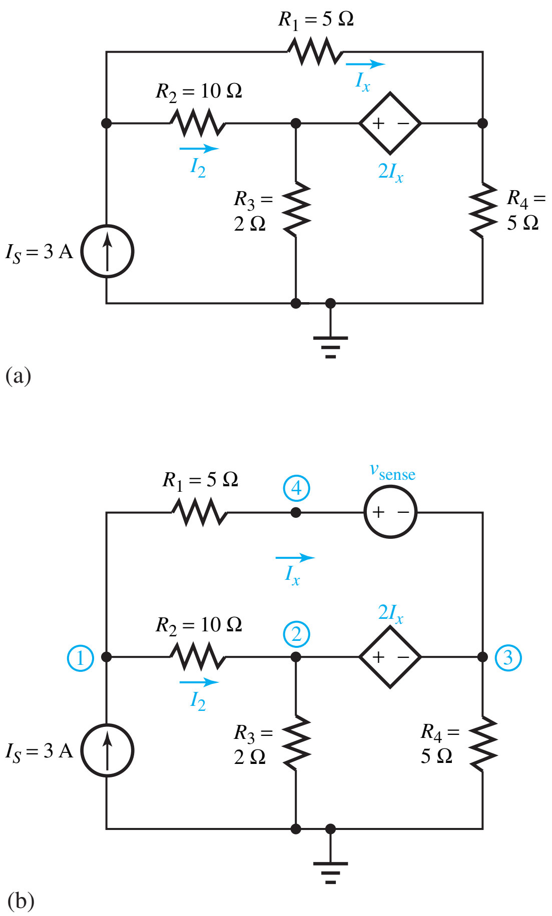


Figure E2.5.1 **(a)** Circuit. **(b)** Redrawn circuit for developing a PSpice program.

2.6 COMPUTER-AIDED CIRCUIT ANALYSIS: MATLAB

This text does not teach MATLAB; it assumes that the student is familiar with it through previous work. Also, the book does not depend on a student having MATLAB. MATLAB, however, provides an enhancement to the learning experience if it is available. If it is not, the problems involving MATLAB can simply be skipped, and the remainder of the text still makes sense. If one wants to get a quick introduction, the book entitled *Getting Started with MATLAB 5* by R. Pratap, listed under Selected Bibliography for Supplemental Reading for Computer-Aided Circuit Analysis, may be a good source.

MATLAB (MATrix LABoratory), a product of The Math Works, Inc., is a software package for high-performance numerical computation and visualization. It is simple, powerful, and for most purposes quite fast with its easy-to-learn and easy-to-use language. It provides an interactive environment with hundreds of built-in functions for technical computation, graphics, and animation. MATLAB also provides easy extensibility with its own high-level programming language.

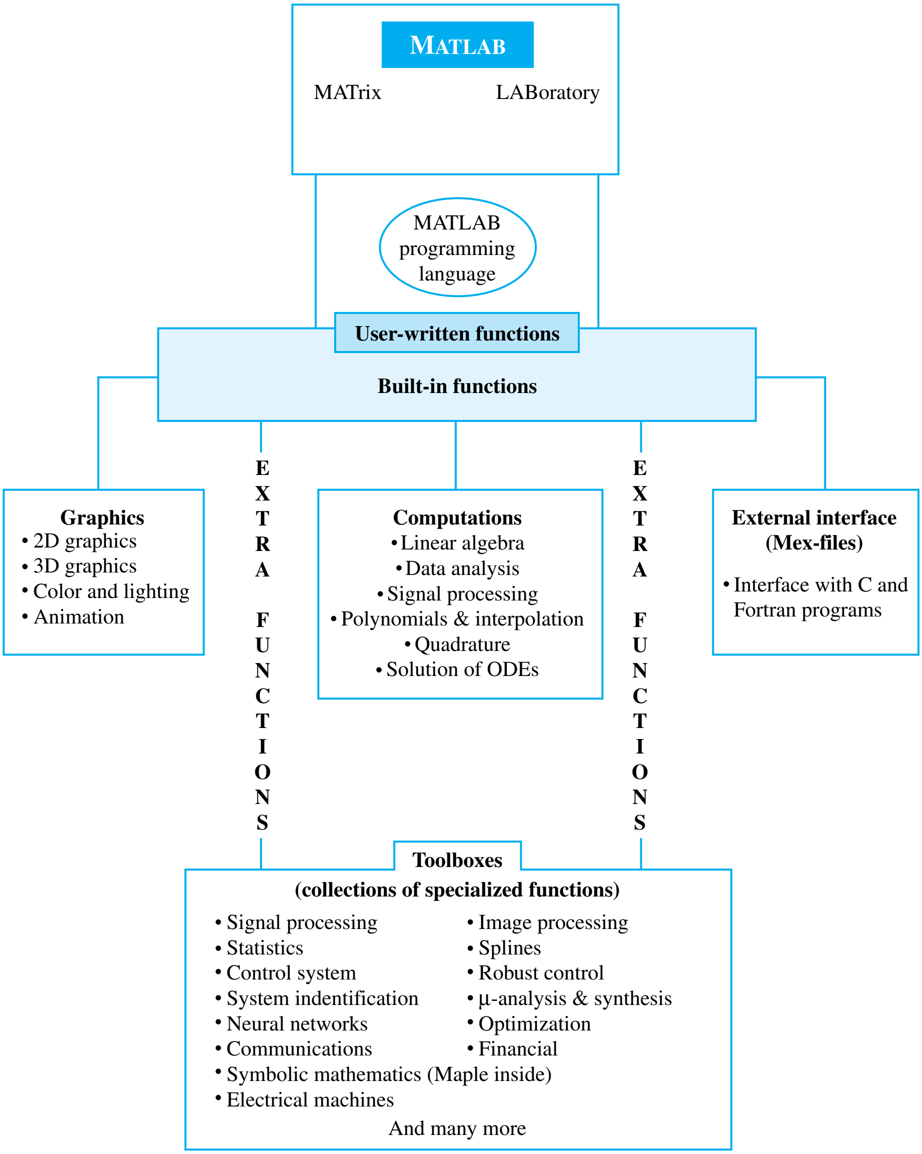


Figure 2.6.1 Schematic diagram of MATLAB's main features.

Figure 2.6.1 illustrates MATLAB's main features. The built-in functions with their state-of-the-art algorithms provide excellent tools for linear algebra computations, data analysis, signal processing, optimization, numerical solution of ordinary differential equations (ODE), numerical integration (Quadrature), and many other types of scientific and engineering computations. Numerous functions are also available for 2D and 3D graphics as well as for animation. Users can also write their own functions, which then behave just like the built-in functions. MATLAB even provides an external interface to run Fortran and C programs. Optional "toolboxes," which are collections of specialized functions for particular applications, are also available. For example, the author of this text has developed “Electrical Machines Toolbox” for the analysis and design of electrical machines.

The MATLAB environment consists of a command window, a figure window, and a platform-dependent edit window, as illustrated in Figure 2.6.2. The command window, which is the main window, is characterized by the MATLAB command prompt >>. All commands, including those for running user-written programs, are typed in this window at the MATLAB prompt. The graphics window or the figure window receives the output of all graphics commands typed in the command window. The user can create as many figure windows as the system memory would allow. The edit window is where one writes, edits, creates, and saves one’s own programs in files called M-files. Most programs that are written in MATLAB are saved as M-files, and all built-in functions in MATLAB are M-files.

Let us now take an illustrative example in circuits to solve a set of simultaneous equations with the use of MATLAB.

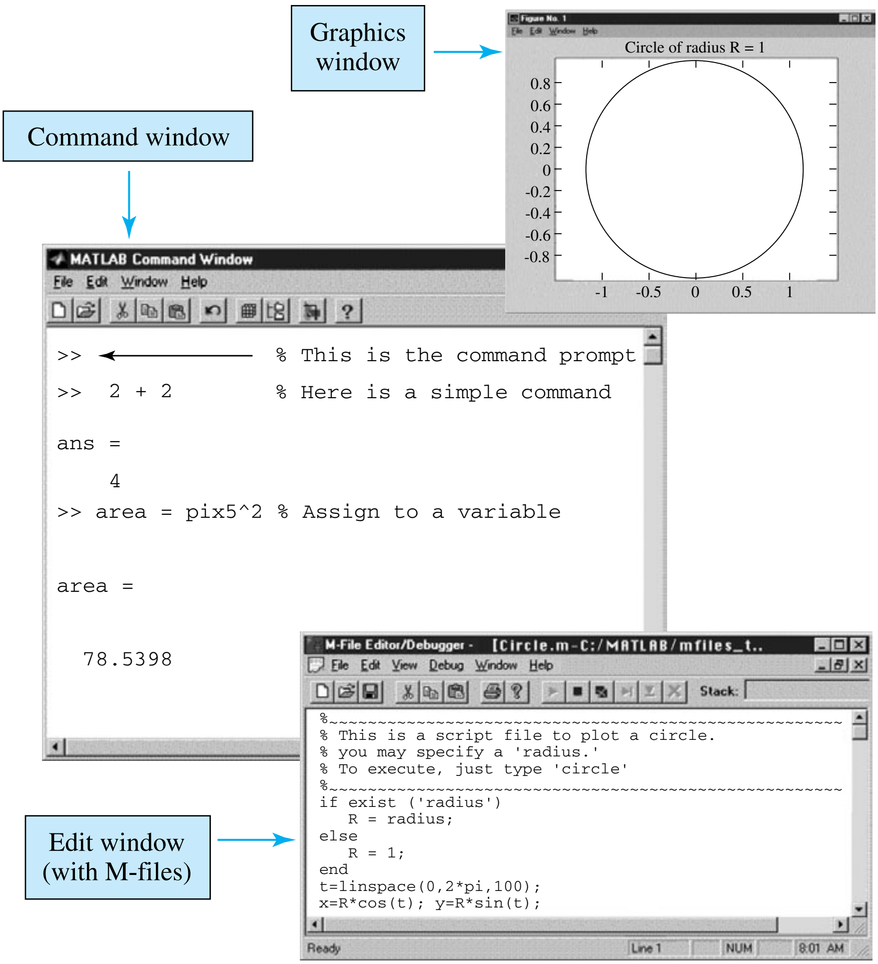


Figure 2.6.2 Illustration of a command window, a figure window, and a platform-dependent edit window in the MATLAB environment.

EXAMPLE 2.6.1

Consider the circuit shown in Figure E2.6.1 and identify the connection equations to be the following:

Node A:	$I_S + I_1 + I_4 = 0;$	Loop 1:	$-V_S + V_1 + V_2 = 0$
Node B:	$-I_1 + I_2 + I_3 = 0;$	Loop 2:	$-V_1 + V_4 - V_3 = 0$
Node C:	$-I_3 - I_4 + I_5 = 0;$	Loop 3:	$-V_2 + V_3 + V_5 = 0$

The element equations are given by

$$V_S = 15; \qquad V_1 = 60I_1; \qquad V_2 = 90I_2$$
$$V_3 = 50I_3; \qquad V_4 = 90I_4; \qquad V_5 = 60I_5$$

Solve these 12 simultaneous equations by using MATLAB and find the voltage across the 50-Ω resistor in the circuit. Also evaluate the total power dissipated in the circuit.

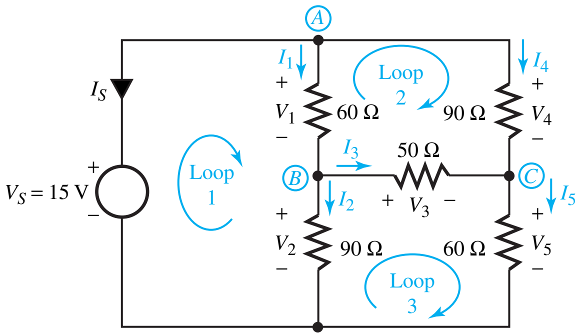


Figure E2.6.1 Circuit for Example 2.6.1.

Solution

The M-file and answers are as follows.

```
function example261
clc

% Given Connection Equations
eqn01 = 'Is + I1 + I4 = 0';
eqn02 = '-I1 + I2 + I3 = 0';
eqn03 = '-I3 - I4 + I5 = 0';

eqn04 = '-Vs + V1 + V2 = 0';
eqn05 = '-V1 - V3 + V4 = 0';
eqn06 = '-V2 + V3 + V5 = 0';

% Element Equations
eqn07 = 'Vs = 15';
eqn08 = 'V1 = 60*I1';
eqn09 = 'V2 = 00*I2';
eqn10 = 'V3 = 50*I3';
eqn11 = 'V4 = 90*I4';
eqn12 = 'V5 = 60*I5';
```

```
% Solve Equations
sol = solve (eqn01, eqn02, eqn03, eqn04, eqn05, eqn06, . . .
             eqn07, eqn08, eqn09, eqn10, eqn11, eqn12, . . .
             'I1, I2, I3, I4, I5, Is, V1, V2, V3, V4, V5, Vs');

% Answers
V3 = eval (sol. V3)
Is = eval (sol. Is)
eval (sol.I1*sol.V1 +sol.I2*sol.V2+sol.I3*sol.V3+sol.I4*sol.V4+
sol.I5*sol.V5)
V3 = 1.2295
IS = -0.2049
ans = 3.0738
```

2.7 LEARNING OBJECTIVES

The learning objectives of this chapter are summarized here, so that the student can check whether he or she has accomplished each of the following.

- Obtaining Thévenin equivalent circuit for a two-terminal (or one-port) network with or without dependent sources.
- Obtaining Norton equivalent circuit for a two-terminal (or one-port) network with or without dependent sources.
- Nodal-voltage method of network analysis, including the concept of a supernode.
- Mesh-current method of network analysis, including the concept of a supermesh.
- Node-voltage and mesh-current equations with controlled sources and their constraint equations.
- Analysis of linear circuits, containing more than one source, by using the principle of superposition.
- Wye–delta transformation for resistive network reduction.
- Computer-aided circuit analysis using SPICE and MATLAB.

2.8 PRACTICAL APPLICATION: A CASE STUDY

Jump Starting a Car

Voltage and current in an electric network are easily measured. They obey Kirchoff's laws, KCL and KVL, and facilitate the monitoring of energy flow. For these reasons, voltage and current are used by engineers in order to describe the state of an electric network.

When a car battery is weak, say 11 V in a 12-V system, in order to jump-start that car, we bring in another car with its engine running and its alternator charging its battery. Let the healthy and strong battery have a voltage of 13 V. According to the recommended practice, one should first connect the positive terminals with the red jumper cable, as shown in Figure 2.8.1, and then complete the circuit between the negative terminals with the aid of the black jumper cable. Note that the negative terminal of any car battery is always connected to its auto chasis.

Applying KVL in Figure 2.8.1, we have

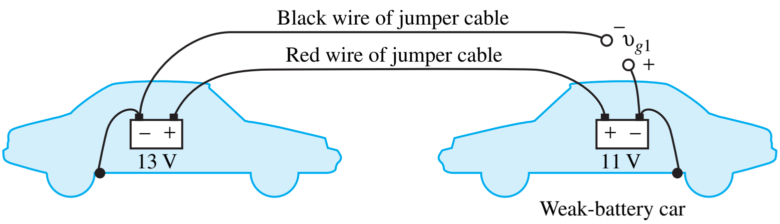


Figure 2.8.1 Jumper cable connections for jump starting a car with a weak battery.

$$v_{g1} - 13 + 11 = 0 \qquad \text{or} \qquad v_{g1} = 2 \text{ V}$$

where v_{g1} is the voltage across the airgap, or the voltage existing between the black jumper cable and the negative terminal of the weak battery.

Now suppose one makes, by mistake, incorrect connections, as shown in Figure 2.8.2. Note that the red jumper cable is connected between the positive terminal of the strong battery and the negative terminal of the weak battery. Application of the KVL now fields

$$v_{g2} - 13 - 11 = 0 \qquad \text{or} \qquad v_{g2} = 24 \text{ V}$$

where v_{g2} is the gap voltage with incorrect connections. With such a large voltage difference, when one tries to complete the black jumper cable connection, it presents a danger to both batteries and to the person making the connections.

Energy to Start an Engine

A simplified circuit model for an automotive starter circuit is shown in Figure 2.8.3. Let the car battery voltage be 12.5 V and let the automobile starter motor draw 60 A when turning over the engine. If the engine starts after 10 seconds, we can easily calculate the power to the starter motor, which is the same as the power out of the battery,

$$P = VI = 12.5 \times 60 = 750 \text{ W}$$

The energy required to start the engine can be computed as

$$W = 750 \times 10 = 7500 \text{ J}$$

Thus, simple circuit models can be used to simulate various physical phenomena of practical interest. They can then be analyzed by circuit-analysis techniques to yield meaningful solutions rather easily.

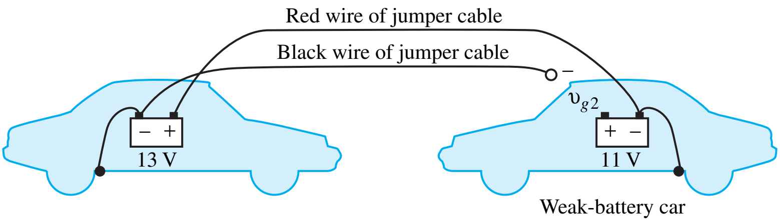


Figure 2.8.2 Incorrect connections for jump starting a car with a weak battery.

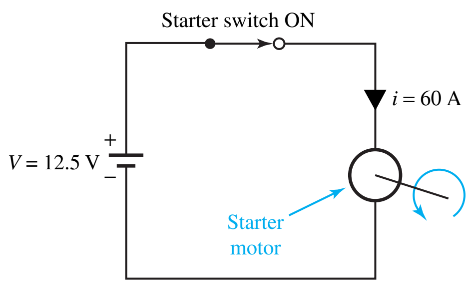


Figure 2.8.3 Simplified circuit model for the automotive starter circuit.

PROBLEMS

2.1.1 (a) Determine the Thévenin and Norton equivalent circuits as viewed by the load resistance R in the network of Figure P2.1.1.

(b) Find the value of R if the power dissipated by R is to be a maximum.

(c) Obtain the value of the power in part (b).

2.1.2 Reduce the circuit of Figure P2.1.2 to a Thévenin and a Norton equivalent circuit.

***2.1.3** Find the Thévenin and Norton equivalent circuits for the configuration of Figure P2.1.3 as viewed from terminals a–b.

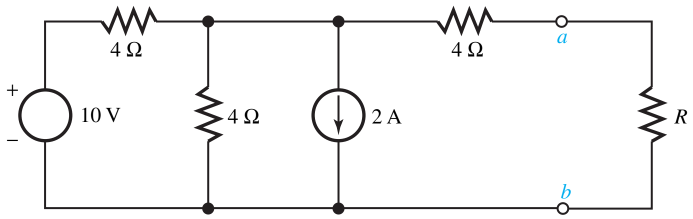


Figure P2.1.1

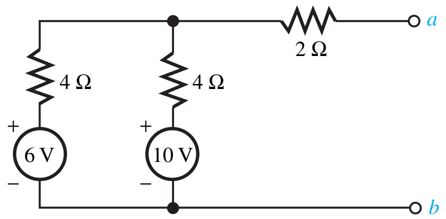


Figure P2.1.2

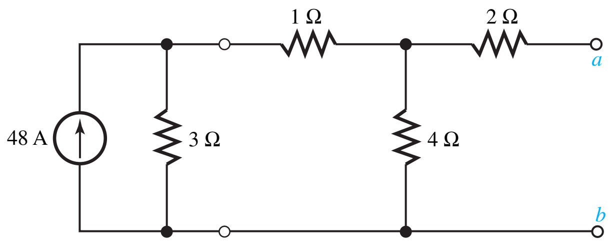


Figure P2.1.3

2.1.4 Obtain the Thévenin and Norton equivalent circuits for the portion of the circuit to the left of terminals *a–b* in Figure P2.1.4, and find the current in the 200-Ω resistance.

2.1.5 Determine the voltage across the 20-Ω load resistance in the circuit of Figure P2.1.5 by the use of the Thévenin equivalent circuit.

2.1.6 Find the current in the 5-Ω resistance of the circuit of Figure P2.1.6 by employing the Norton equivalent circuit.

***2.1.7** Obtain the voltage across the 3-kΩ resistor of the circuit (transistor amplifier stage) given in Figure P2.1.7 by the use of the Thévenin equivalent circuit.

2.1.8 Reduce the circuit of Figure P2.1.8 to a Thévenin and a Norton equivalent circuit with respect to terminals *a–b*.

2.1.9 (a) Redraw the circuit in Figure P2.1.9 by replacing the portion to the left of terminals *a–b* with its Thévenin equivalent.

(b) Redraw the circuit of Figure P2.1.9 by replacing the portion to the right of terminals *a′–b′* with its Thévenin equivalent.

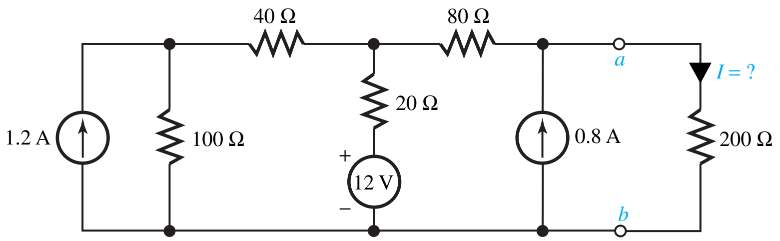


Figure P2.1.4

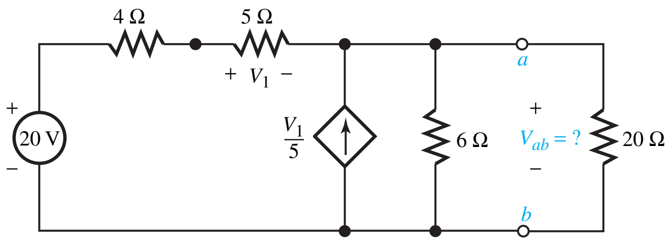


Figure P2.1.5

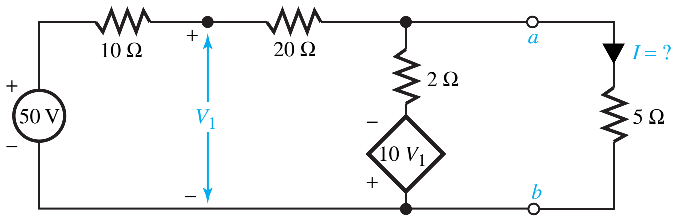


Figure P2.1.6

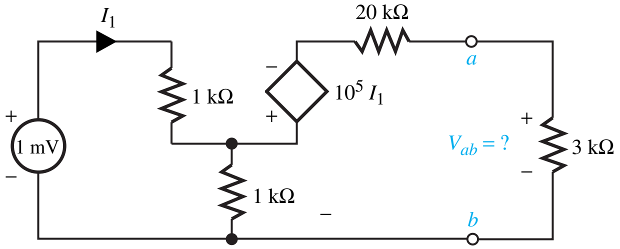


Figure P2.1.7

2.1.10 (a) Consider the Wheatstone bridge circuit given in Figure P2.1.10(a) and find the Thévenin equivalent with respect to terminals *a–b*.

(b) Suppose a source with resistance is connected across *a–b*, as shown in Figure P2.1.10(b). Then find the current I_{ab}.

2.2.1 In the circuit given in Figure P2.2.1, determine the current I through the 2-Ω resistor by (a) the nodal-voltage method, and (b) mesh-current analysis.

2.2.2 Consider the circuit of Figure P2.2.2 and rearrange it such that only one loop equation is required to solve for the current I.

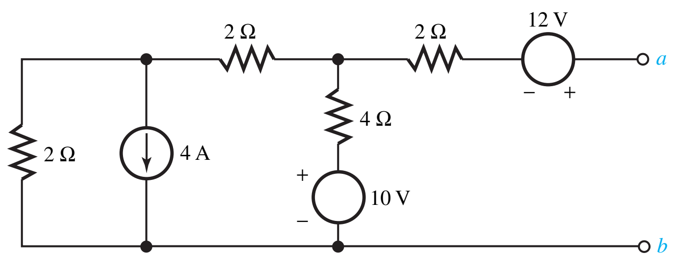


Figure P2.1.8

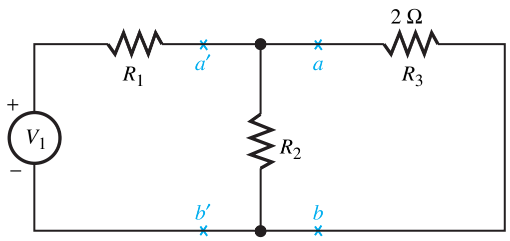
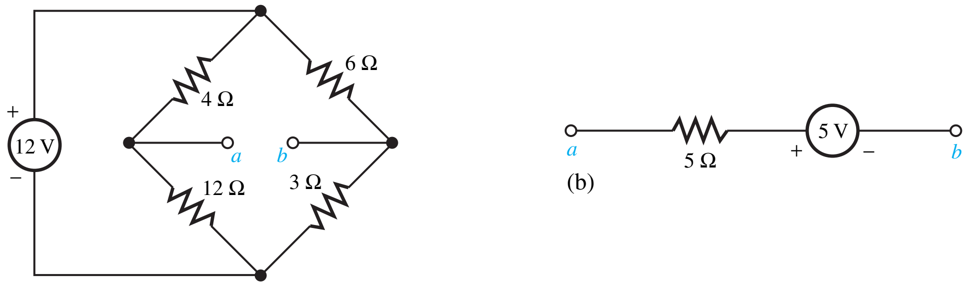


Figure P2.1.9



(a)

Figure P2.1.10

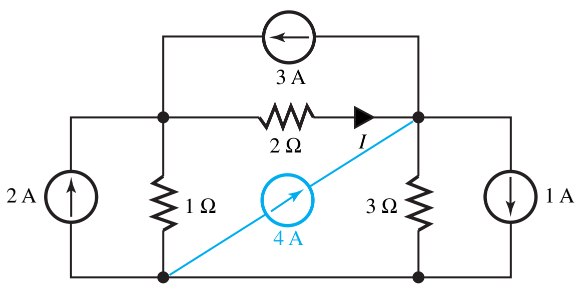


Figure P2.2.1

2.2.3 Use the node-voltage method to find the current I through the 5-Ω resistor of the circuit of Figure P2.2.3.

2.2.4 Use the node-voltage method to determine the voltage across the 12-Ω resistor of the circuit given in Figure P2.2.4. Verify by mesh analysis.

2.2.5 Determine the current I through the 10-Ω resistor of the circuit of Figure P2.2.5 by employing the node-voltage method. Check by mesh analysis.

***2.2.6** (a) Find the voltage across the 8-A current source in the circuit of Figure P2.2.6 with the use of nodal analysis.

(b) Determine the current in the 0.5-Ω resistor of the circuit by mesh analysis.

2.2.7 By using the mesh-current method, determine the voltage across the 1-A current source of the circuit of Figure P2.2.7, and verify by nodal analysis.

2.2.8 Find the current I_1 through the 20-Ω resistor of the circuit of Figure P2.2.8 by both mesh and nodal analyses.

2.2.9 Determine the voltage V in the circuit of Figure P2.2.9 by nodal analysis and verify by mesh analysis.

2.2.10 Find the current I in the circuit of Figure P2.2.10 by mesh analysis and verify by nodal analysis.

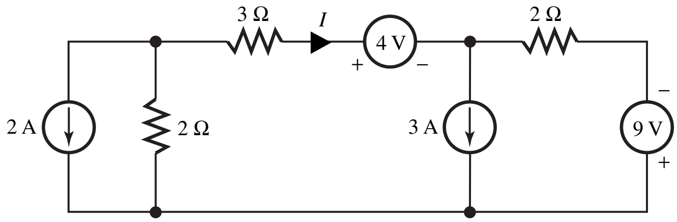


Figure P2.2.2

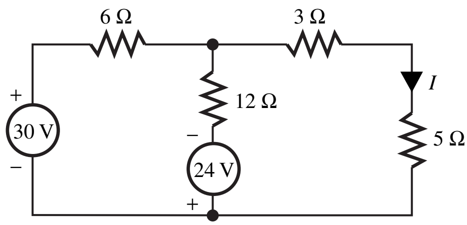


Figure P2.2.3

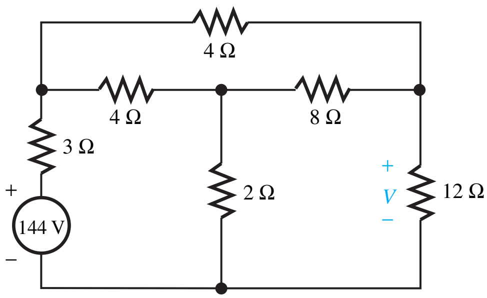


Figure P2.2.4

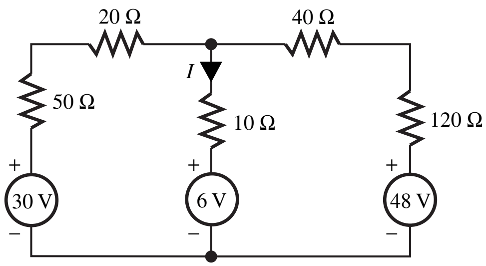


Figure P2.2.5

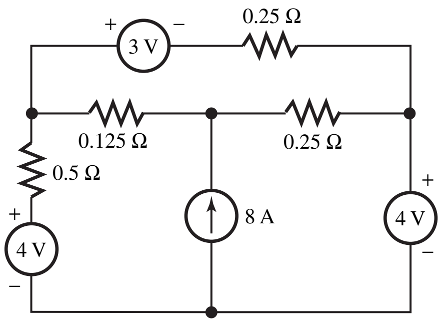


Figure P2.2.6

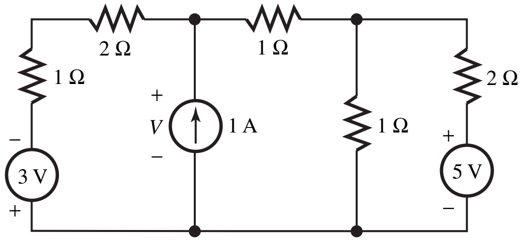


Figure P2.2.7

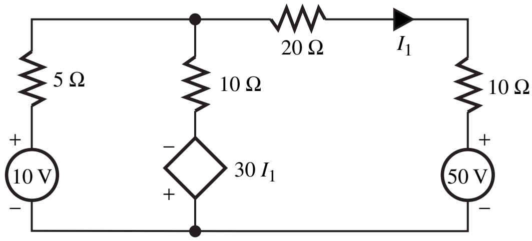


Figure P2.2.8

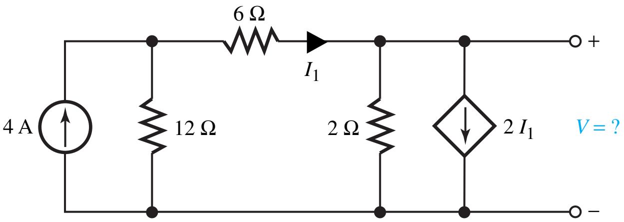


Figure P2.2.9

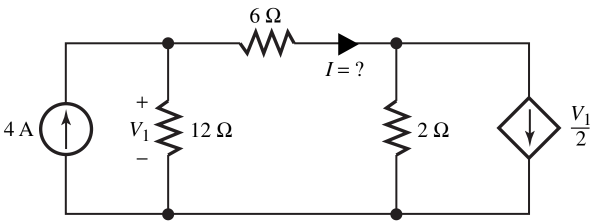


Figure P2.2.10

2.2.11 For the network of Figure P2.2.11, find the nodal voltages V_1, V_2, and V_3 by means of nodal analysis, using the concept of a supernode. Verify by mesh-current analysis.

***2.2.12** Use nodal analysis and the supernode concept to find V_2 in the circuit shown in Figure P2.2.12. Verify by mesh-current analysis, by using source transformation and by using the concept of a supermesh.

2.2.13 Use mesh-current analysis and the supermesh concept to find V_0 in the circuit of Figure P2.2.13. Verify by nodal analysis.

2.2.14 For the network shown in Figure P2.2.14, find V_x across the 3-Ω resistor by using mesh current analysis. Verify by means of nodal analysis.

2.3.1 Consider the circuit of Problem 2.2.1 and find the current I through the 2-Ω resistor by the principle of superposition.

2.3.2 Solve Problem 2.2.3 by the application of superposition.

2.3.3 Solve Problem 2.2.5 by the application of superposition.

2.3.4 Solve Problem 2.2.6 by the application of superposition.

2.3.5 Solve Problem 2.2.7 by the application of superposition.

***2.3.6** Solve Problem 2.2.8 by the application of superposition.

2.4.1 Show that Equations (2.4.1) and (2.4.2) are true.

***2.4.2** Determine R_S in the circuit of Figure P2.4.2 such that it is matched at terminals a–b, and find the power delivered by the voltage source.

2.4.3 Find the power delivered by the source in the circuit given in Figure P2.4.3. Use network reduction by wye–delta transformation.

2.5.1 Develop and execute a PSpice program to analyze the circuit shown in Figure P2.5.1 to evaluate the node voltages and the current through each element.

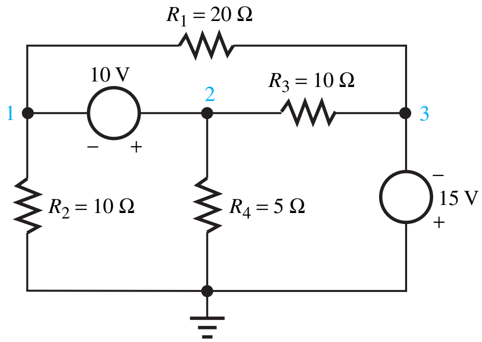


Figure P2.2.11

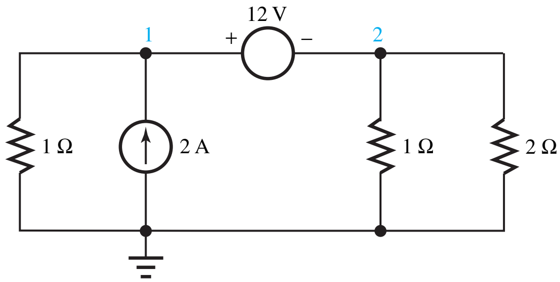


Figure P2.2.12

Figure P2.2.13

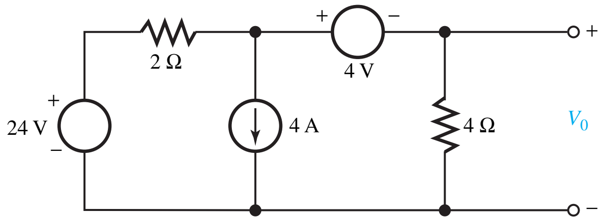
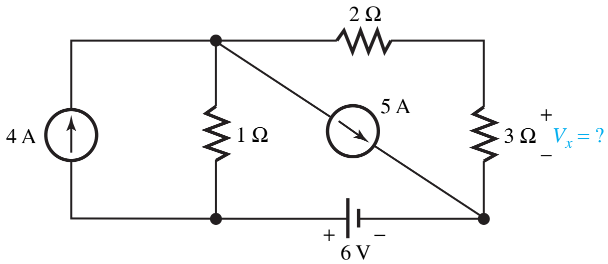


Figure P2.2.14



2.5.2 Develop and execute a PSpice program to find the node voltages and the current through each element of the circuit given in Figure P2.5.2.

***2.5.3** For the circuit shown in Figure P2.5.3, develop and execute a PSpice program to obtain the node voltages and the current through each element.

2.5.4 For the circuit given in Figure P2.5.4, develop and execute a PSpice program to solve for the node voltages.

2.5.5 Write and execute a PSpice program to analyze the resistor bridge circuit shown in Figure P2.5.5 to solve for the node voltages and the voltage-source current. Then find the voltage across the 50-Ω resistor and the total power supplied by the source.

2.6.1 The current through a 2.5-mH indicator is a damped sine given by $i(t) = 10e^{-500t} \sin 2000t$. With the aid of MATLAB, plot the waveforms of the inductor current $i(t)$, with voltage $v(t) = L\,di/dt$, power $p(t) = vi$, and energy $w(t) = \int_0^t p(\tau)\,d\tau$. Starting at $t = 0$, the plots should include at least one cycle and at least 20 points per cycle.

***2.6.2** An interface circuit consisting of R_1 and R_2 is designed between the source and the load, as illustrated in Figure P2.6.2 such that the load sees a Thévenin resistance of 50 Ω between terminals C and D, while simultaneously the source sees a load resistance of 300 Ω between A and B. Using MATLAB, find R_1, and R_2.

Hint: solve the two nonlinear equations given by

$$\frac{(R_1 + 300)R_2}{R_1 + 300 + R_2} = 50; \qquad R_1 + \frac{50R_2}{R_2 + 50} = 300$$

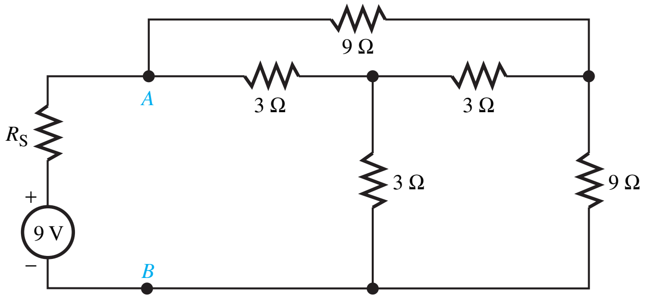


Figure P2.4.2

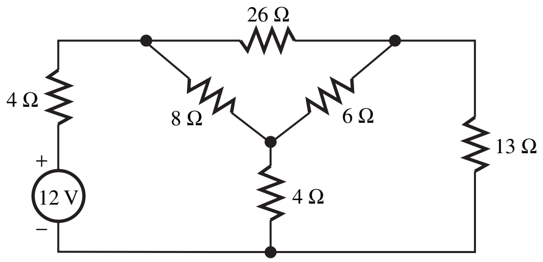


Figure P2.4.3

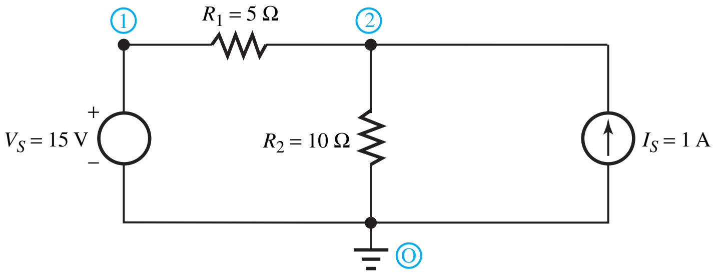


Figure P2.5.1

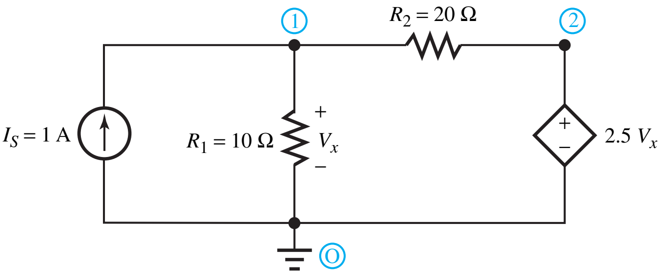


Figure P2.5.2

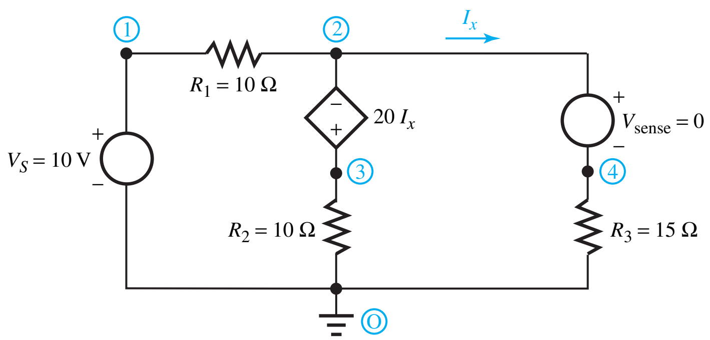


Figure P2.5.3

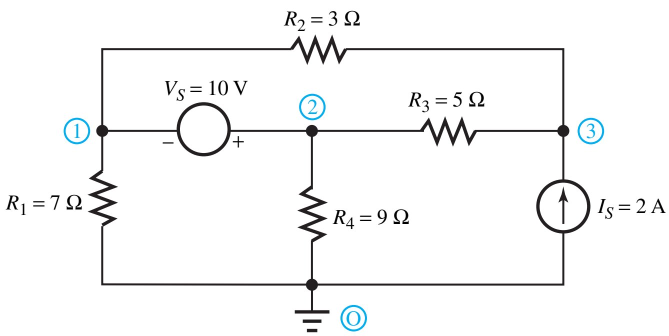


Figure P2.5.4

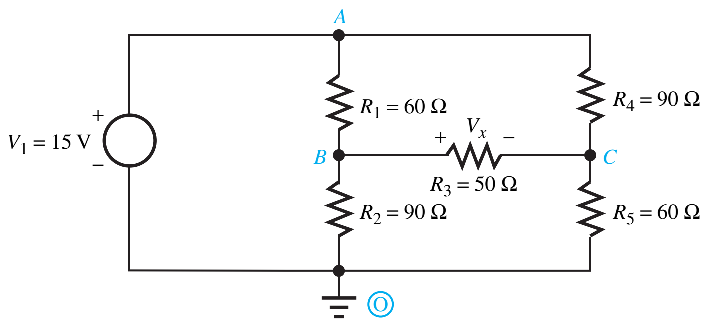


Figure P2.5.5

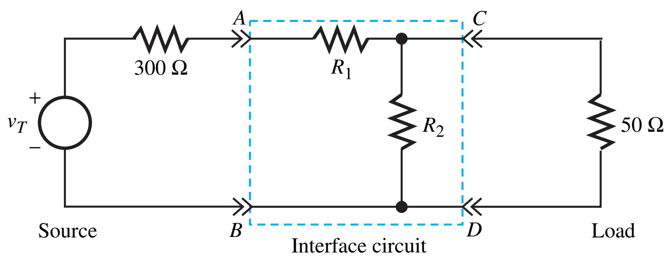


Figure P2.6.2